

# Examples of principal nested torii (general $d$ ) – Part II

*Eduardo García-Portugués, Huiling Le, James S. Marron, and Andrew Wood*

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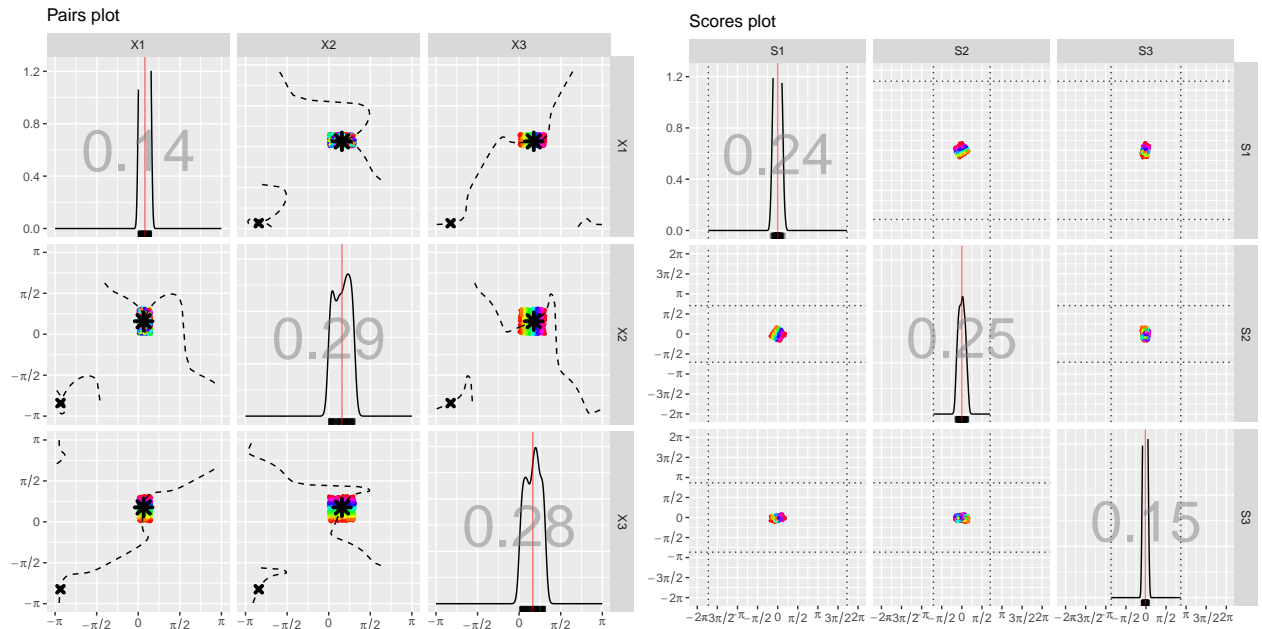
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## Examples in $d = 3$

### Uniform-like

#### Uniform vertical band

## Reduction to dimension  $d = 3$ . Time: 1.316 seconds.  
 ## Reduction to dimension  $d = 2$ . Time: 0.393 seconds.  
 ## Reduction to dimension  $d = 1$ . Time: 0.951 seconds.

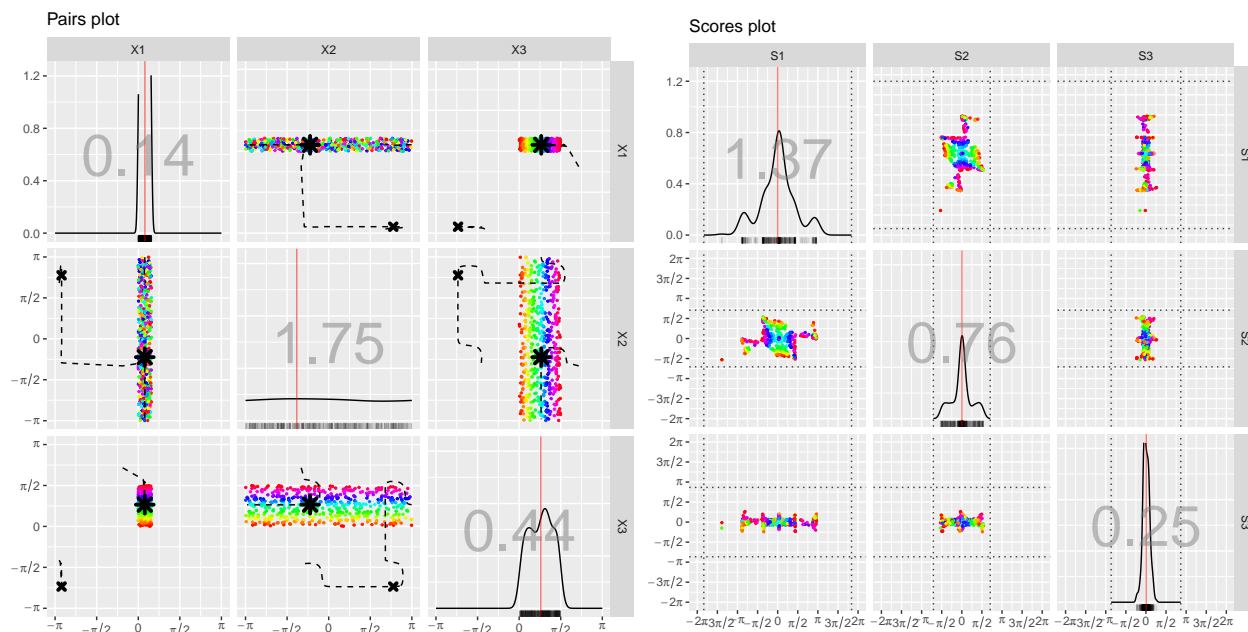


## Uniform plane

## Reduction to dimension  $d = 3$ . Time: 1.445 seconds.

## Reduction to dimension  $d = 2$ . Time: 0.264 seconds.

## Reduction to dimension  $d = 1$ . Time: 0.682 seconds.



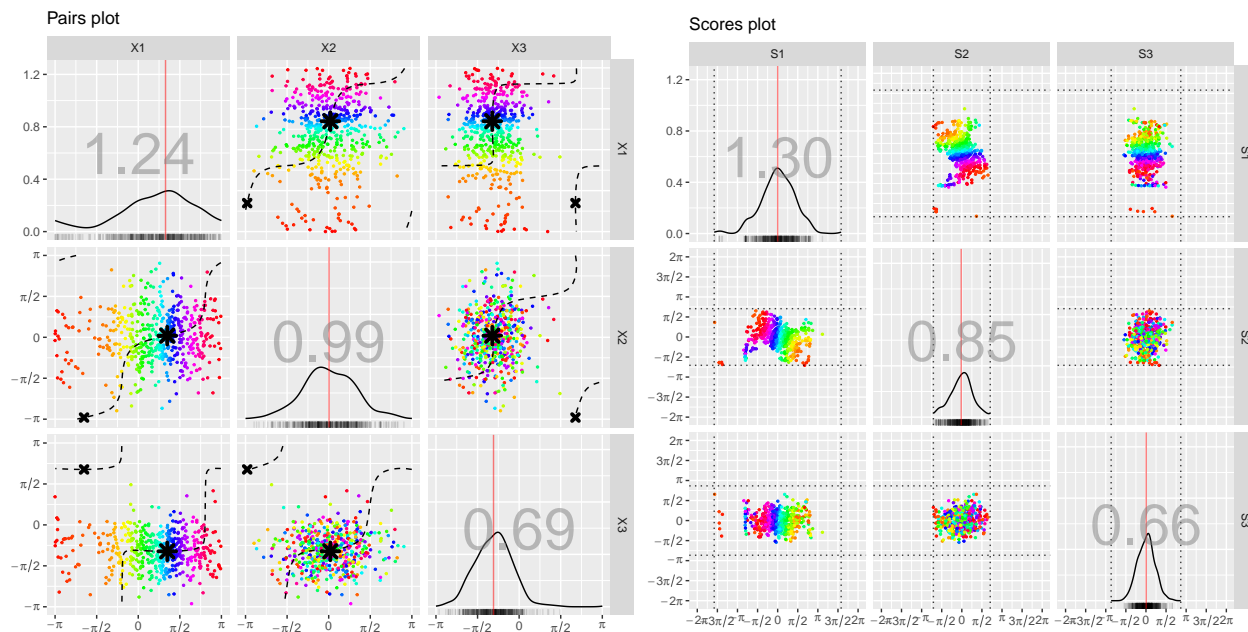
## Gaussian-like

### Non-isotropic

## Reduction to dimension  $d = 3$ . Time: 0.653 seconds.

## Reduction to dimension  $d = 2$ . Time: 0.218 seconds.

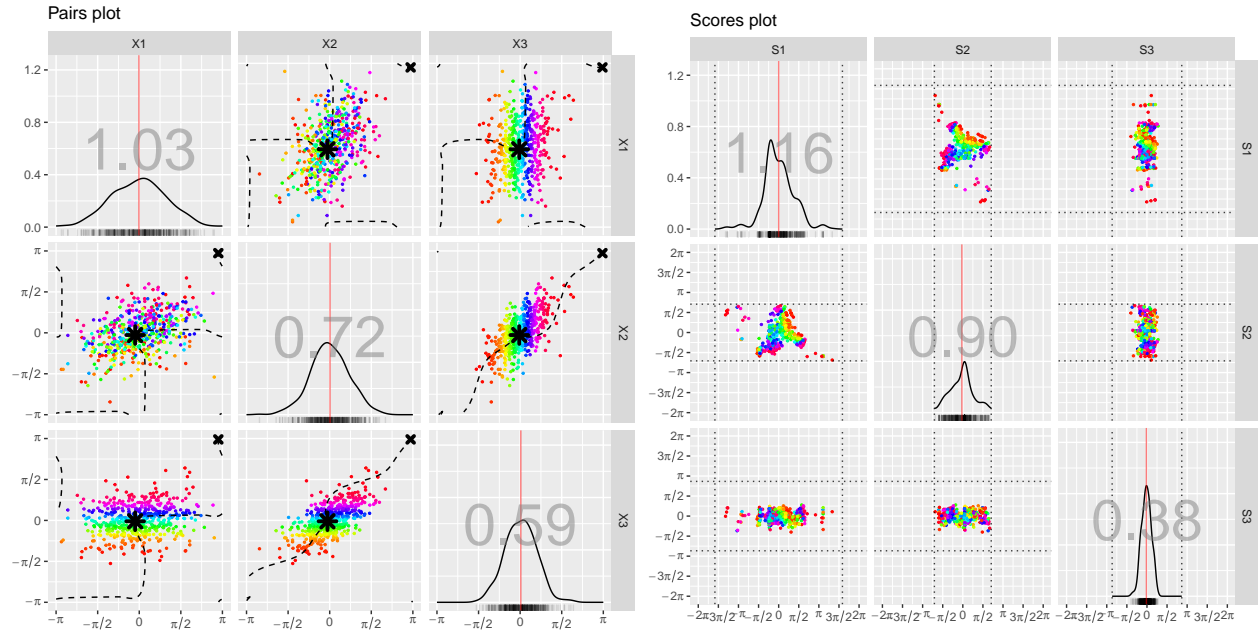
## Reduction to dimension  $d = 1$ . Time: 0.604 seconds.





## Rotated

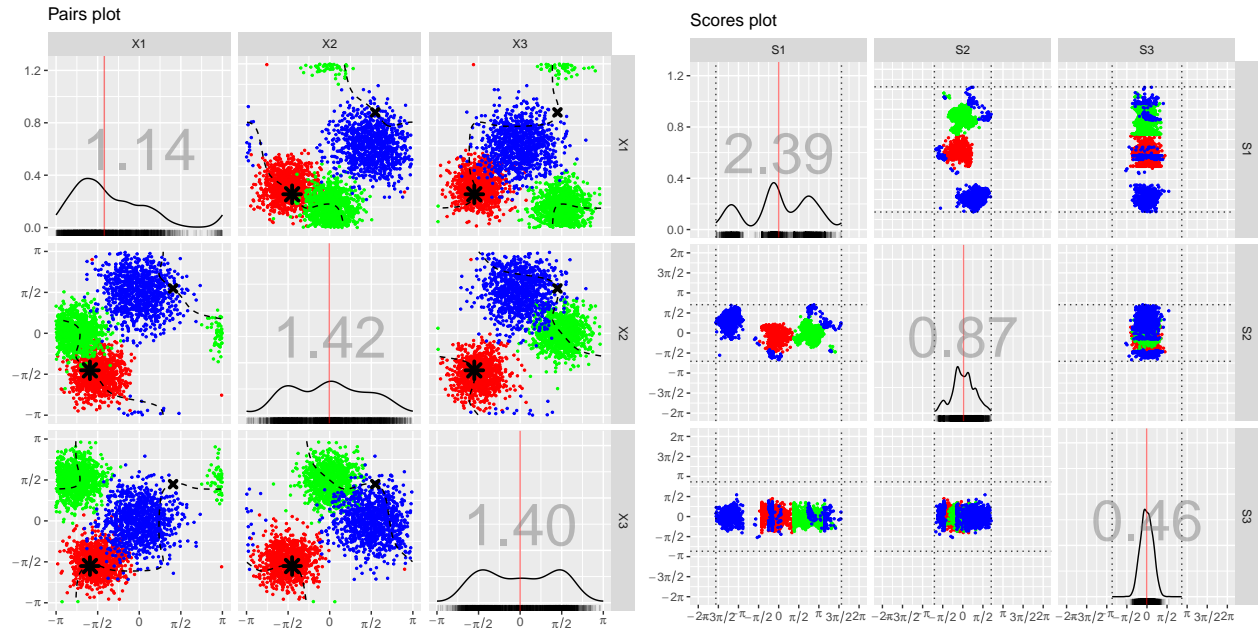
## Reduction to dimension  $d = 3$ . Time: 0.808 seconds.  
 ## Reduction to dimension  $d = 2$ . Time: 0.246 seconds.  
 ## Reduction to dimension  $d = 1$ . Time: 0.9 seconds.



## Clusters

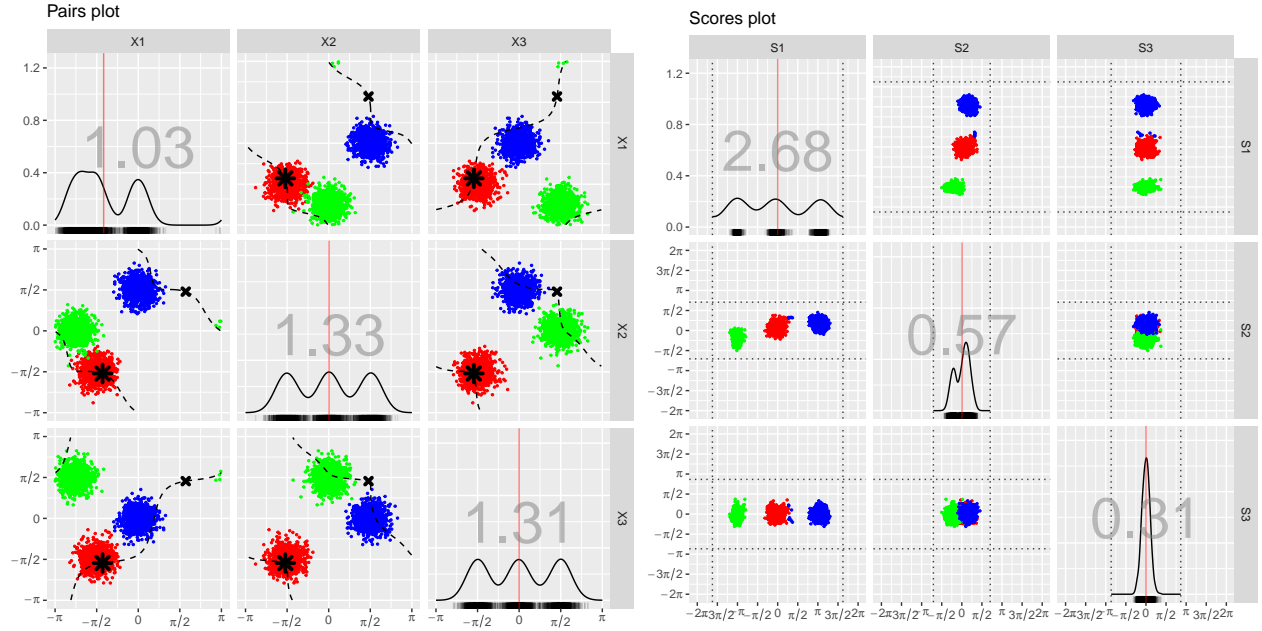
### Clusters 1

## Reduction to dimension  $d = 3$ . Time: 3.371 seconds.  
 ## Reduction to dimension  $d = 2$ . Time: 0.794 seconds.  
 ## Reduction to dimension  $d = 1$ . Time: 1.883 seconds.



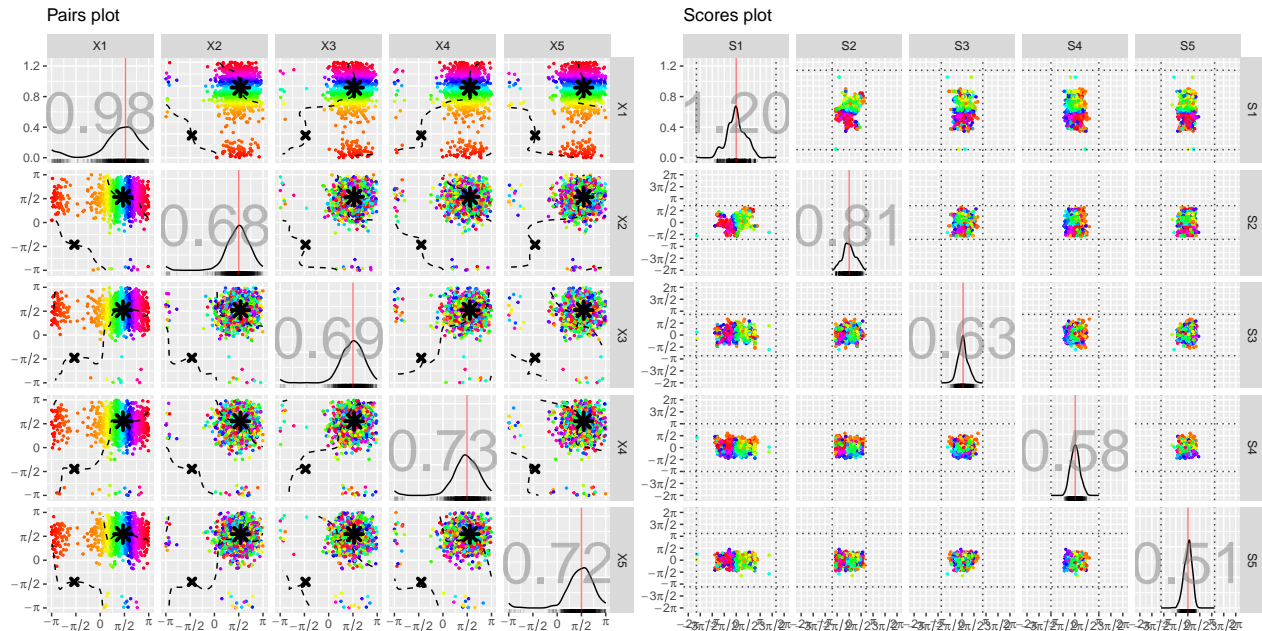
## Clusters 2

```
## Reduction to dimension d = 3. Time: 2.855 seconds.
## Reduction to dimension d = 2. Time: 0.883 seconds.
## Reduction to dimension d = 1. Time: 1.873 seconds.
```

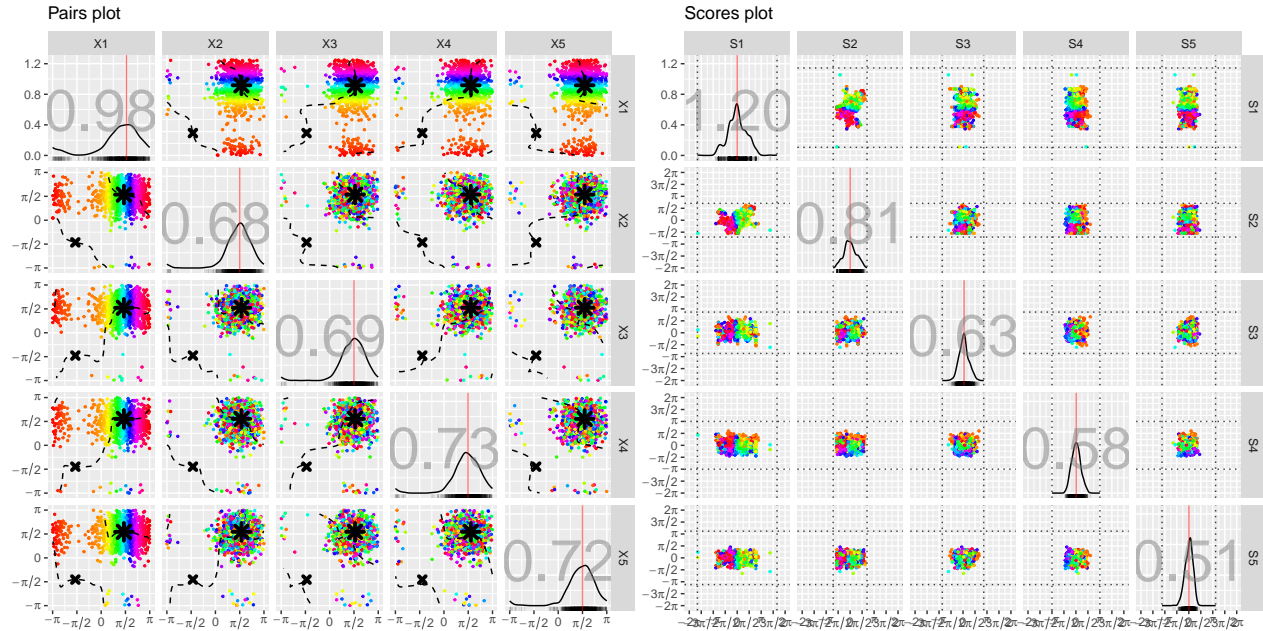


## Examples in $d > 3$

```
## Reduction to dimension d = 5. Time: 7.264 seconds.
## Reduction to dimension d = 4. Time: 2.515 seconds.
## Reduction to dimension d = 3. Time: 1.226 seconds.
## Reduction to dimension d = 2. Time: 0.238 seconds.
## Reduction to dimension d = 1. Time: 0.658 seconds.
```



```
## Reduction to dimension d = 5. Time: 7.545 seconds.
## Reduction to dimension d = 4. Time: 2.371 seconds.
## Reduction to dimension d = 3. Time: 1.144 seconds.
## Reduction to dimension d = 2. Time: 0.252 seconds.
## Reduction to dimension d = 1. Time: 0.67 seconds.
```



## $\lambda$ effect

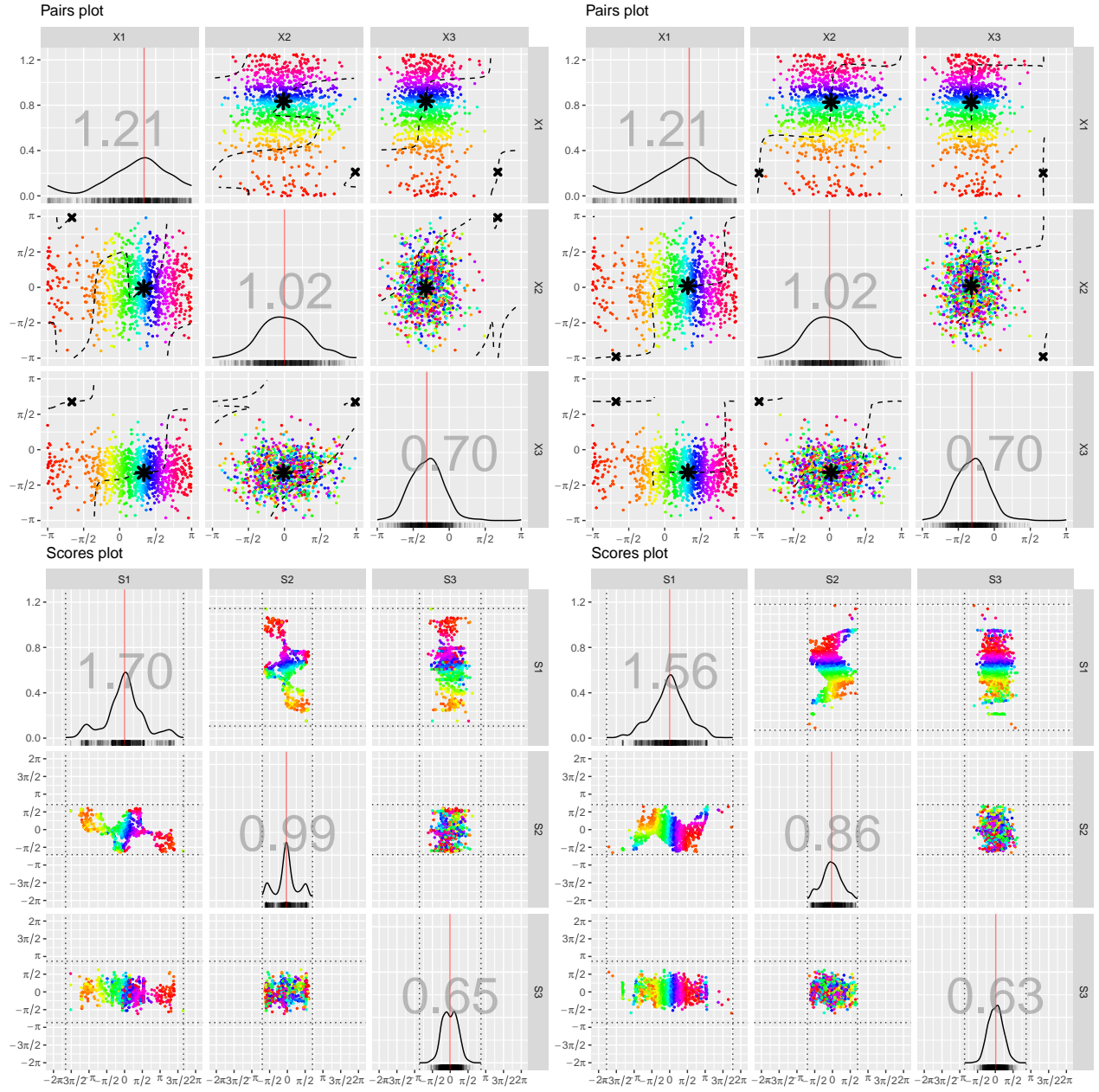
Legend for the next plots:

- *Left* plots corresponds to fits with  $\lambda = 0$ .
- *Right* to fits with  $\lambda = 1$ .

## Non-isotropic Gaussian

```
## Reduction to dimension d = 3. Time: 1.633 seconds.
## Reduction to dimension d = 2. Time: 0.375 seconds.
## Reduction to dimension d = 1. Time: 0.952 seconds.

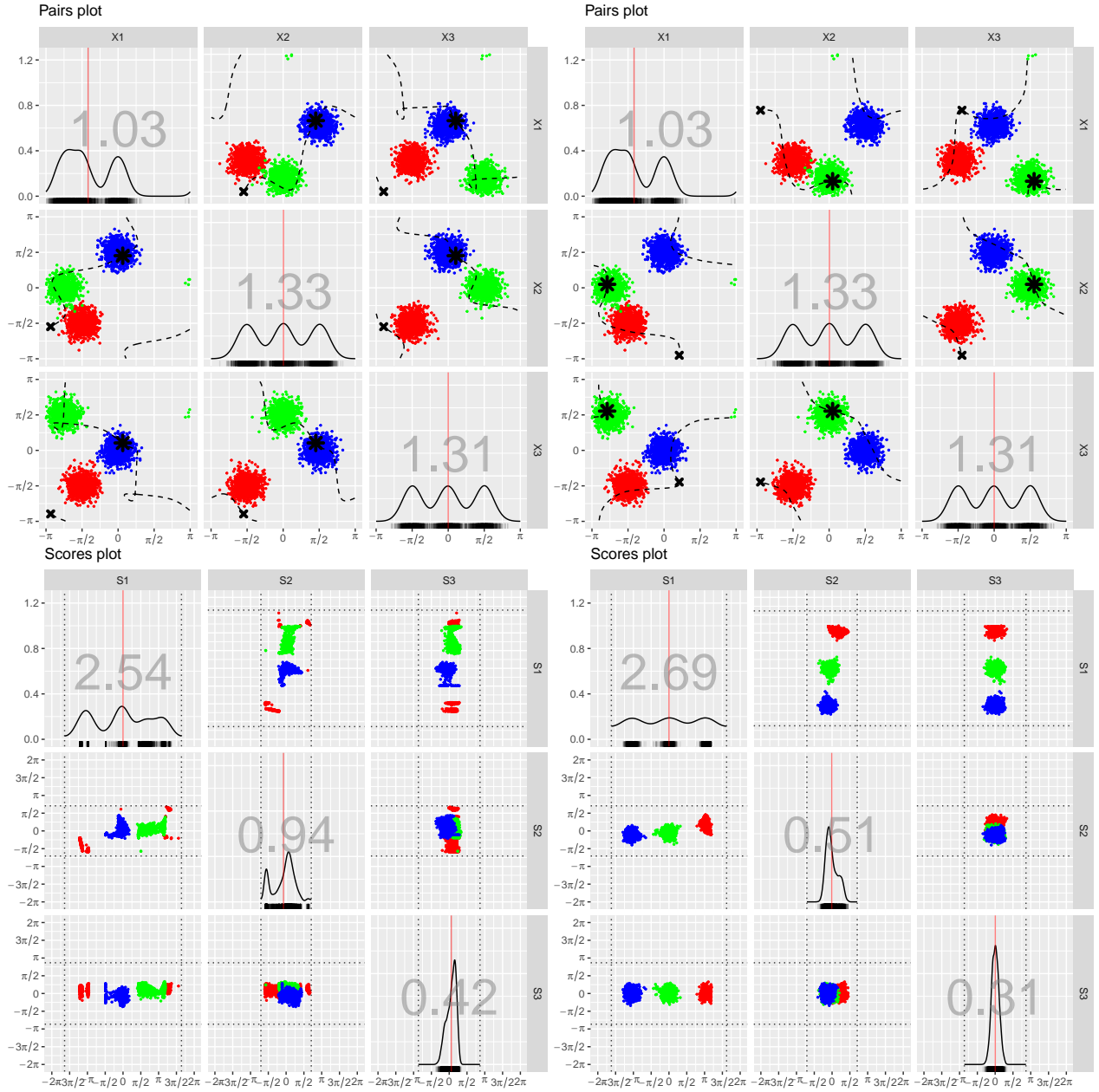
## Reduction to dimension d = 3. Time: 1.214 seconds.
## Reduction to dimension d = 2. Time: 0.311 seconds.
## Reduction to dimension d = 1. Time: 0.723 seconds.
```



## Clusters 2

```
## Reduction to dimension d = 3. Time: 3.54 seconds.
## Reduction to dimension d = 2. Time: 0.745 seconds.
## Reduction to dimension d = 1. Time: 1.682 seconds.

## Reduction to dimension d = 3. Time: 2.787 seconds.
## Reduction to dimension d = 2. Time: 0.684 seconds.
## Reduction to dimension d = 1. Time: 1.628 seconds.
```

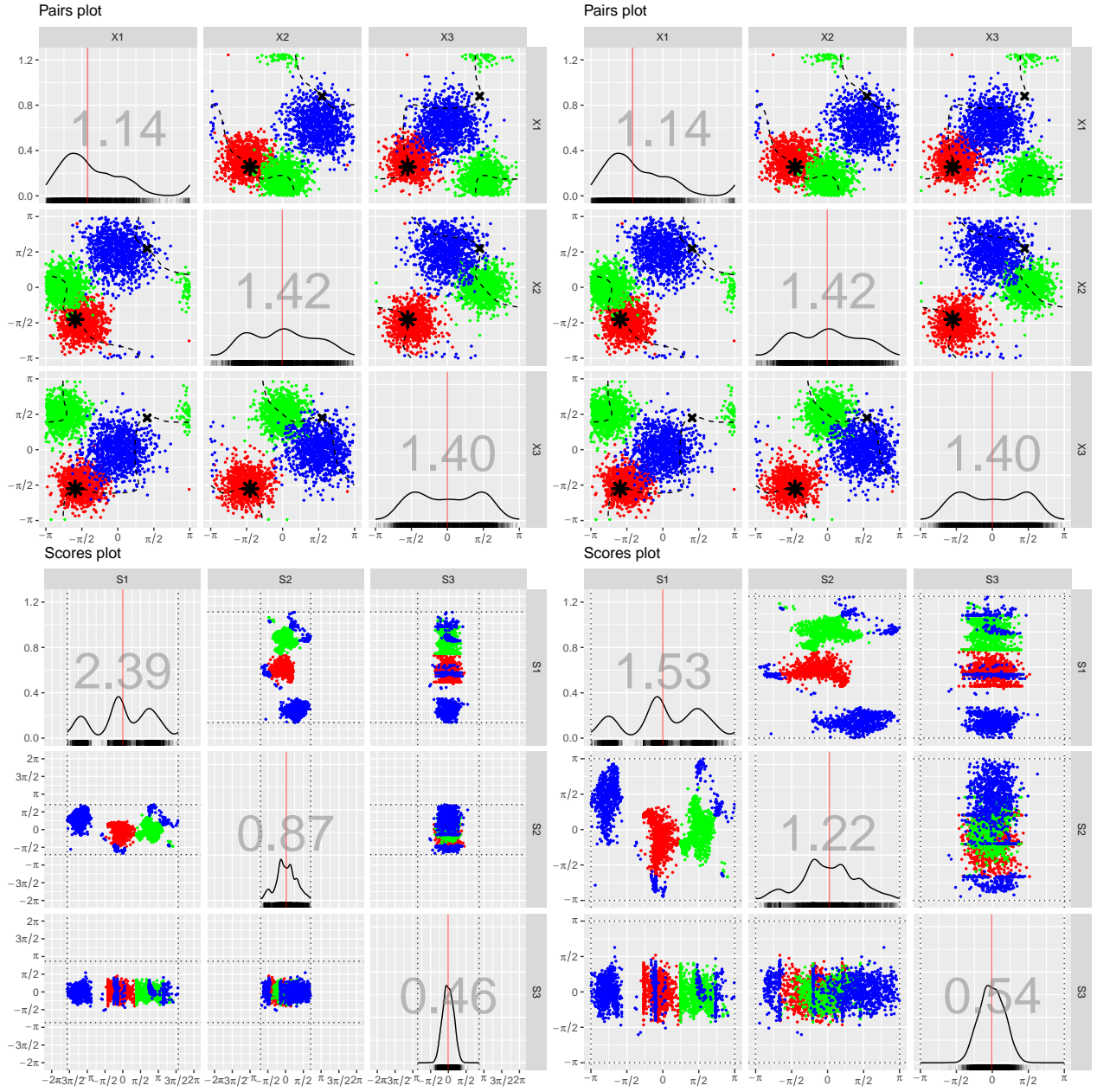


## Issue 1: re-scaling vs. not re-scaling the raw scores

```
## Reduction to dimension d = 3. Time: 3.803 seconds.
## Reduction to dimension d = 2. Time: 0.834 seconds.
## Reduction to dimension d = 1. Time: 1.812 seconds.

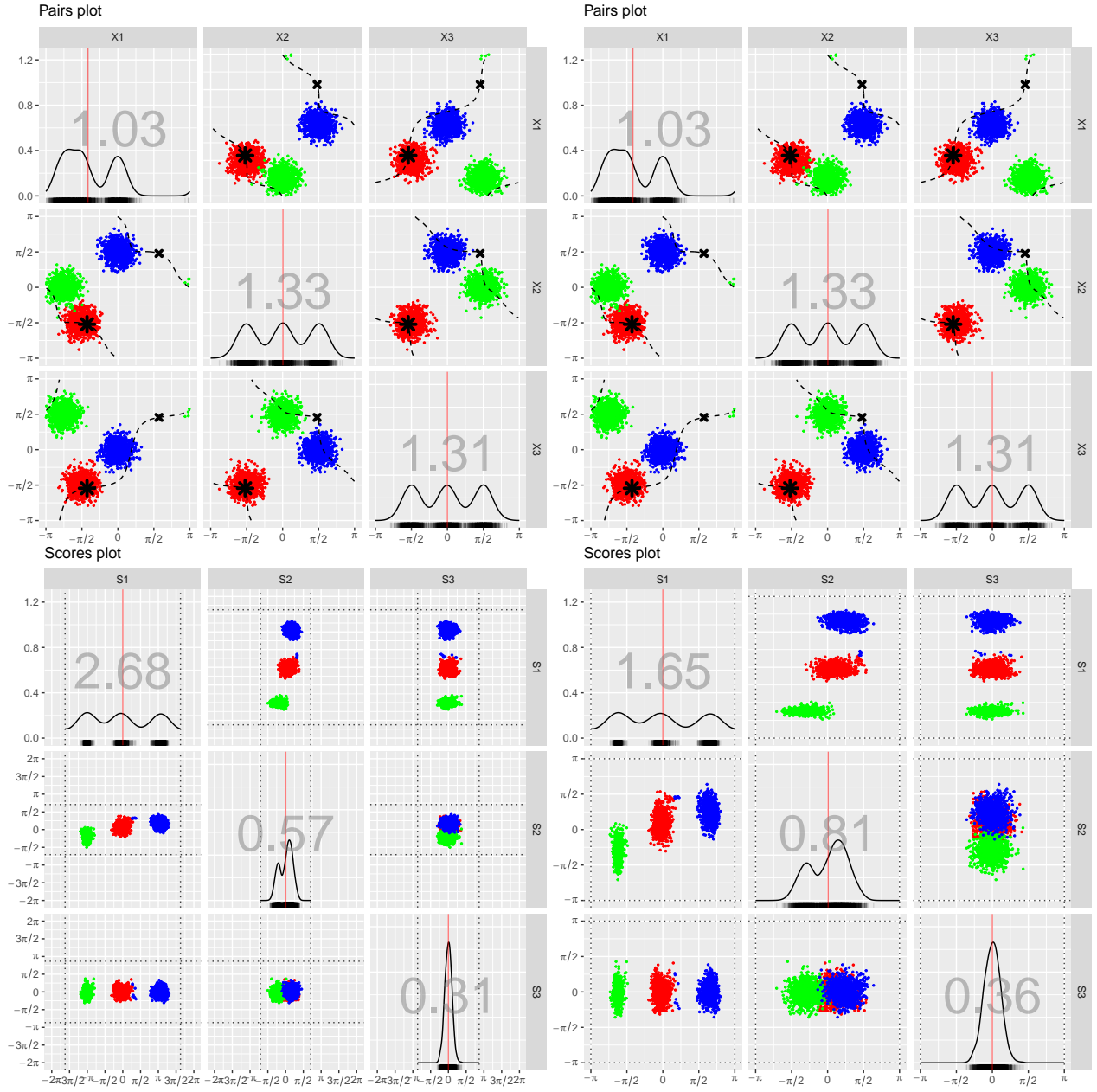
## Reduction to dimension d = 3. Time: 3.179 seconds.
## Reduction to dimension d = 2. Time: 0.714 seconds.
## Reduction to dimension d = 1. Time: 1.634 seconds.
```

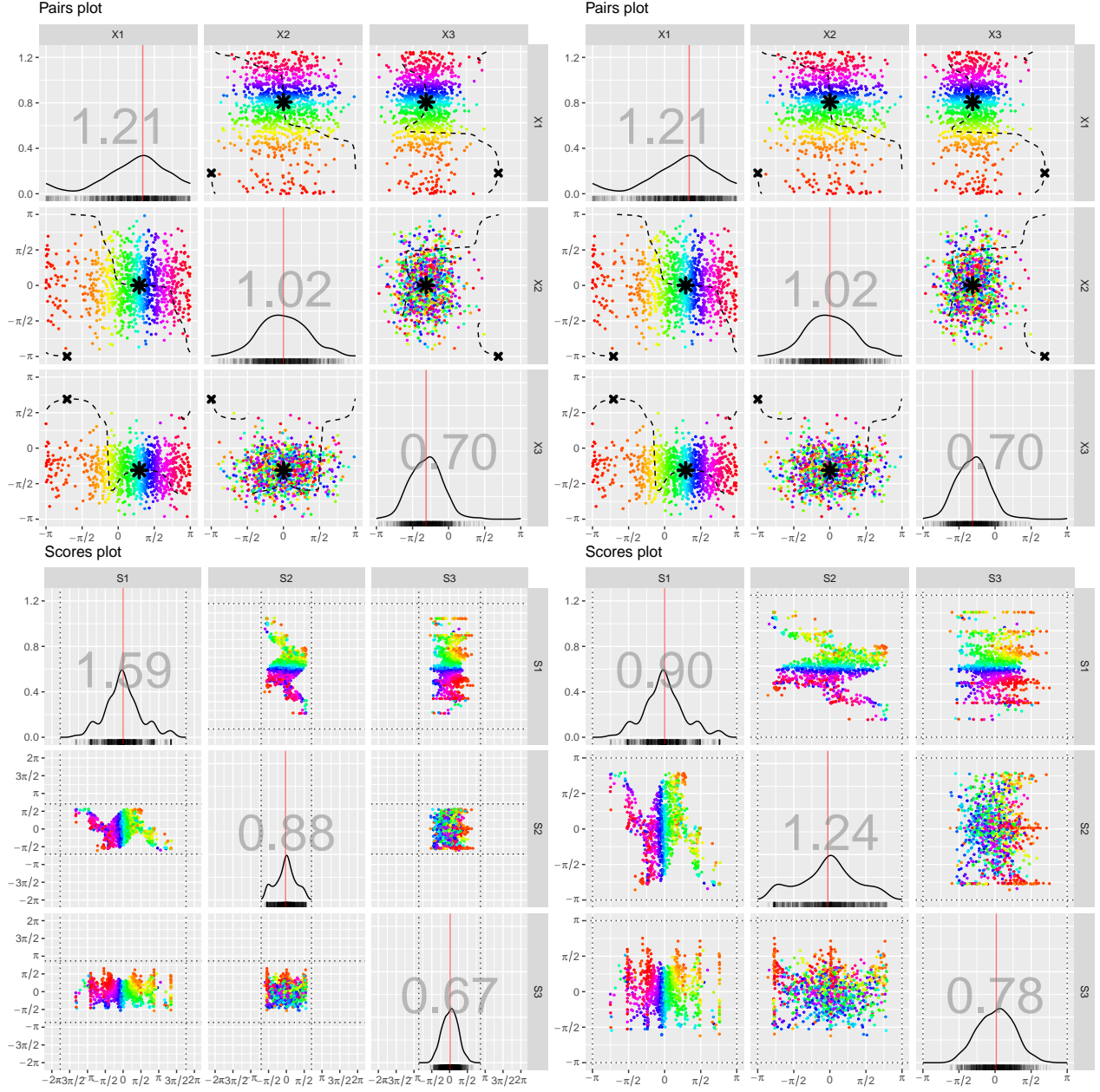




```
## Reduction to dimension d = 3. Time: 2.991 seconds.
## Reduction to dimension d = 2. Time: 0.837 seconds.
## Reduction to dimension d = 1. Time: 1.875 seconds.

## Reduction to dimension d = 3. Time: 2.989 seconds.
## Reduction to dimension d = 2. Time: 0.874 seconds.
## Reduction to dimension d = 1. Time: 1.917 seconds.
```





## Issue 2: computational scalability

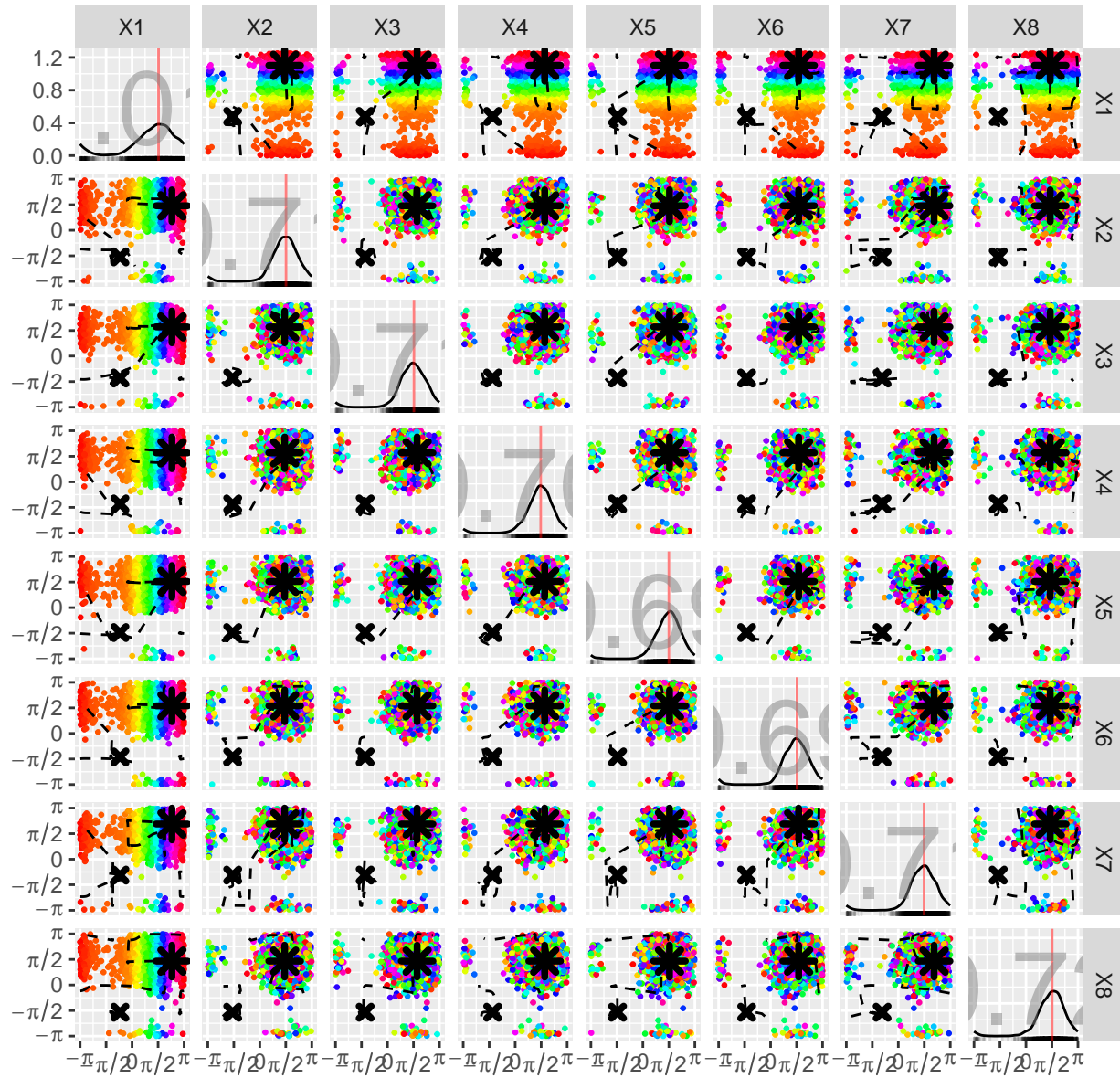
**Gradient computation** is the bottleneck of the optimization procedure. In principle, it is possible to obtain it analytically using matrix differential calculus, but before attacking that problem we have to be completely sure about the objective function.

```
## Reduction to dimension d = 8. Time: 156.213 seconds.
## Reduction to dimension d = 7. Time: 70.283 seconds.
## Reduction to dimension d = 6. Time: 37.436 seconds.
## Reduction to dimension d = 5. Time: 14.017 seconds.
## Reduction to dimension d = 4. Time: 6.788 seconds.
## Reduction to dimension d = 3. Time: 1.657 seconds.
```

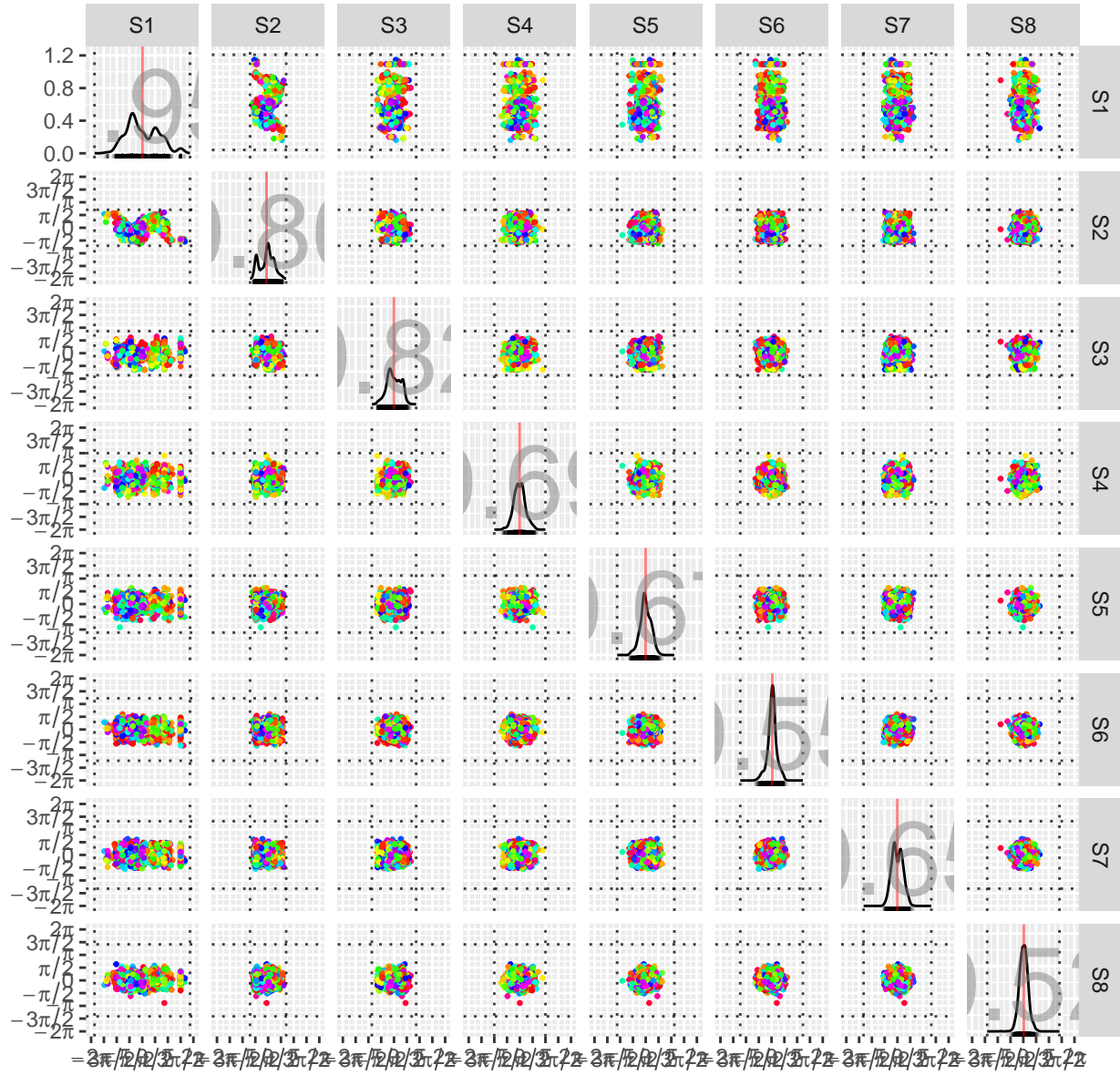


## Reduction to dimension  $d = 2$ . Time: 0.567 seconds.  
## Reduction to dimension  $d = 1$ . Time: 1.213 seconds.

Pairs plot

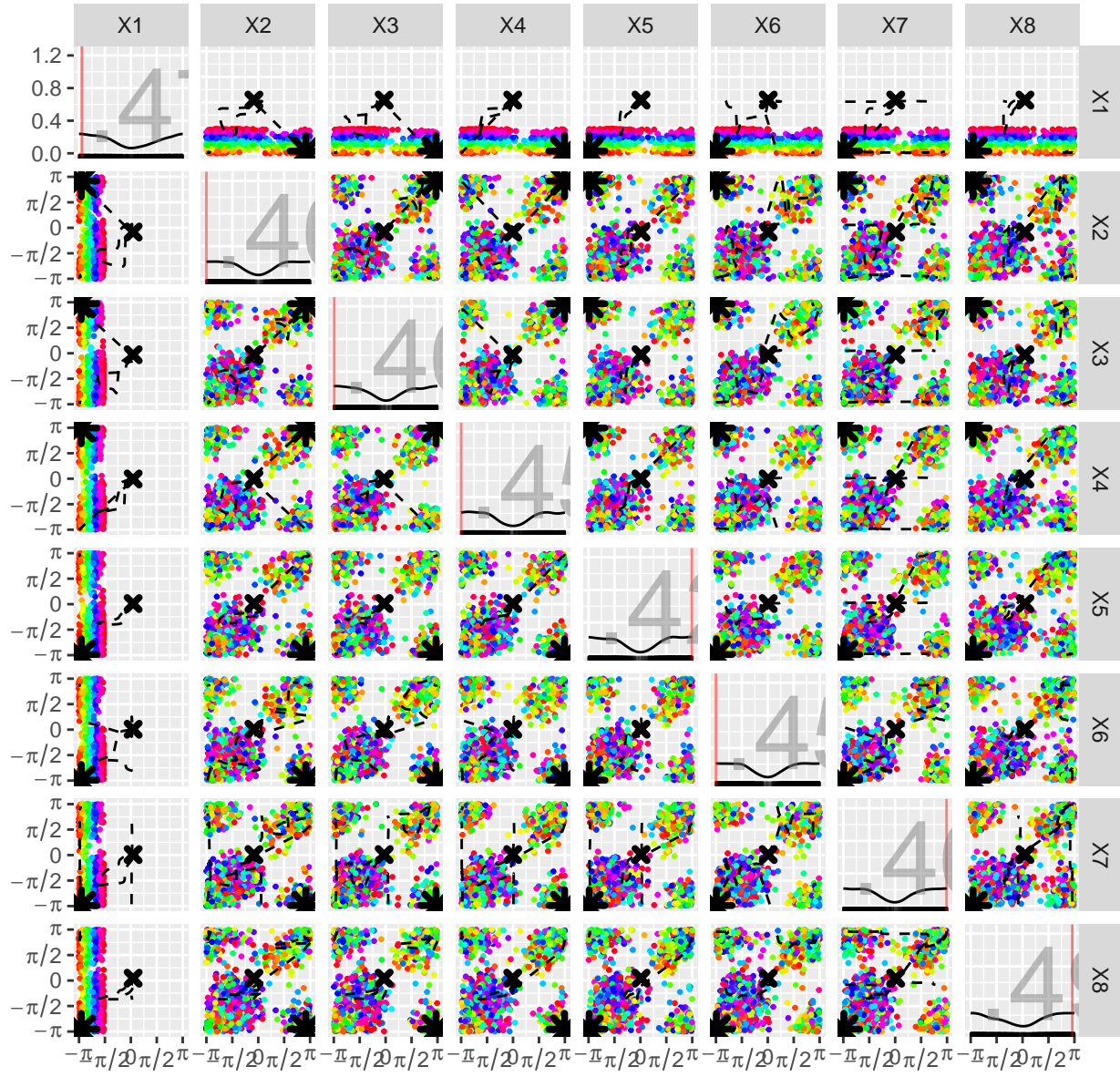


Scores plot



```
## Reduction to dimension d = 8. Time: 247.27 seconds.
## Reduction to dimension d = 7. Time: 79.83 seconds.
## Reduction to dimension d = 6. Time: 55.478 seconds.
## Reduction to dimension d = 5. Time: 18.621 seconds.
## Reduction to dimension d = 4. Time: 9.956 seconds.
## Reduction to dimension d = 3. Time: 3.225 seconds.
## Reduction to dimension d = 2. Time: 0.633 seconds.
## Reduction to dimension d = 1. Time: 1.465 seconds.
```

Pairs plot



Scores plot

