RICE

# MULTIPLICATIVE MODELING

# AND

# MULTISCALE QUEUING ANALYSIS

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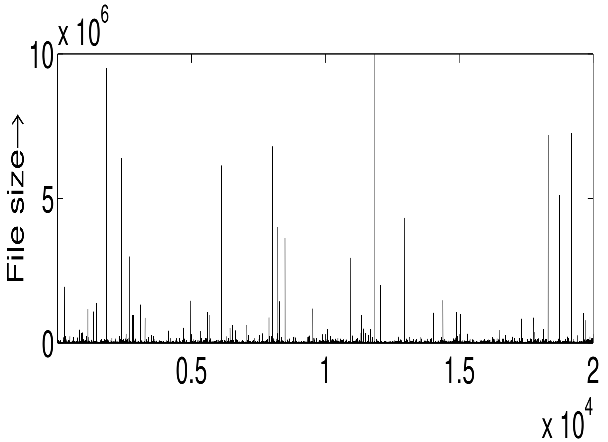
## What we expect from models

| | |
|---|---|
| Simplicity, parsimony | SYNTHESIS |
| Novelty | NEW INTUITION |
| Tractable analysis | COME UP WITH ANSWERS |
| Estimation | ENHANCE STATISTICS SKILLS |
| Extensions | ENLARGE POOL OF MODELS |

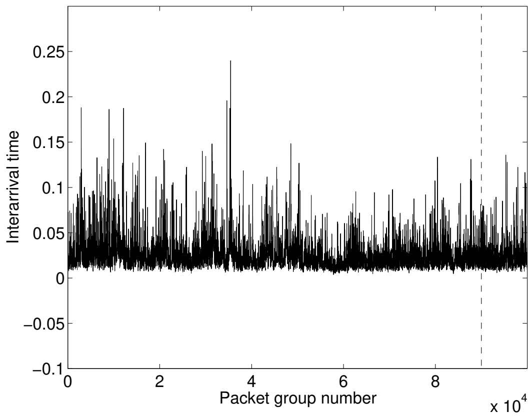
# Burstiness everywhere

Web Requests:



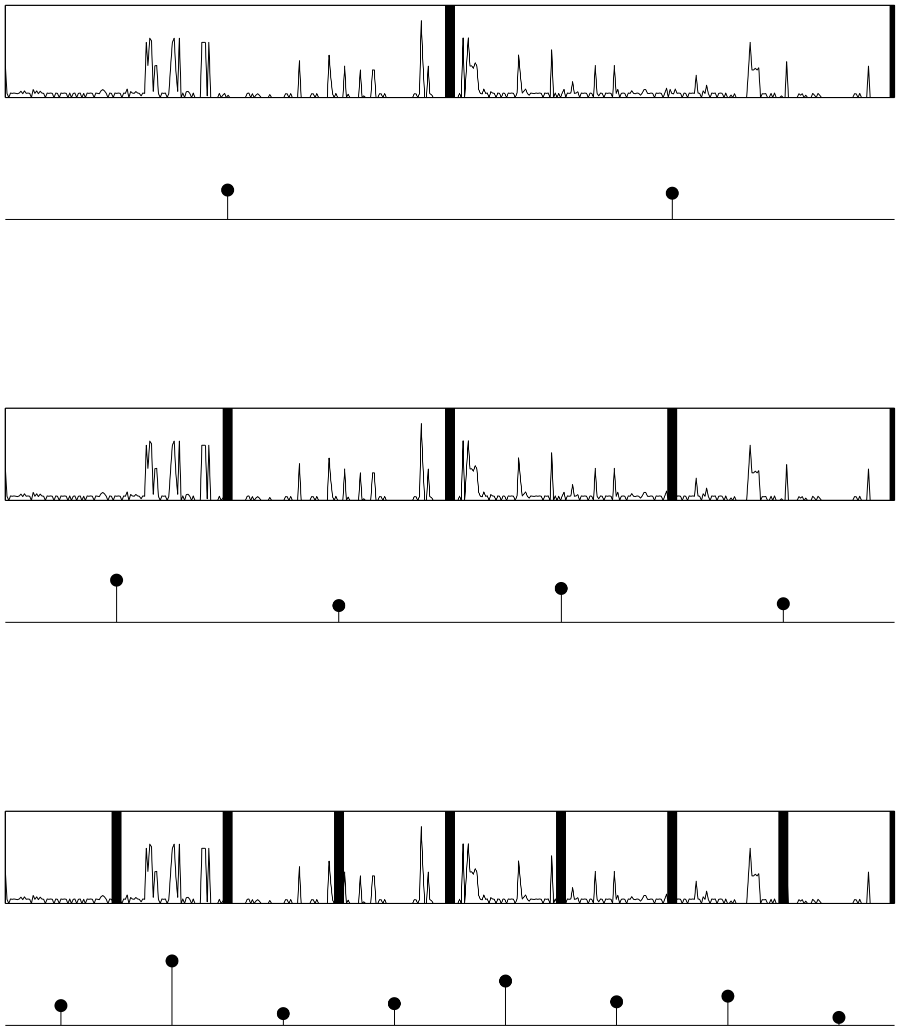

File sizes

Traffic loads at gateway




Interarrival times

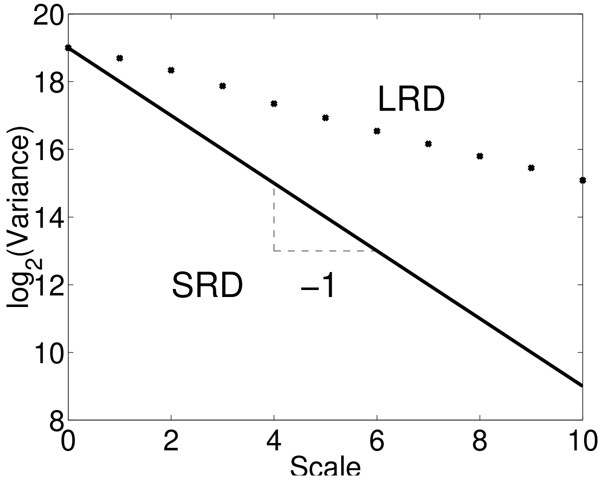
# Multiscale Modeling (LRD)



LRD:

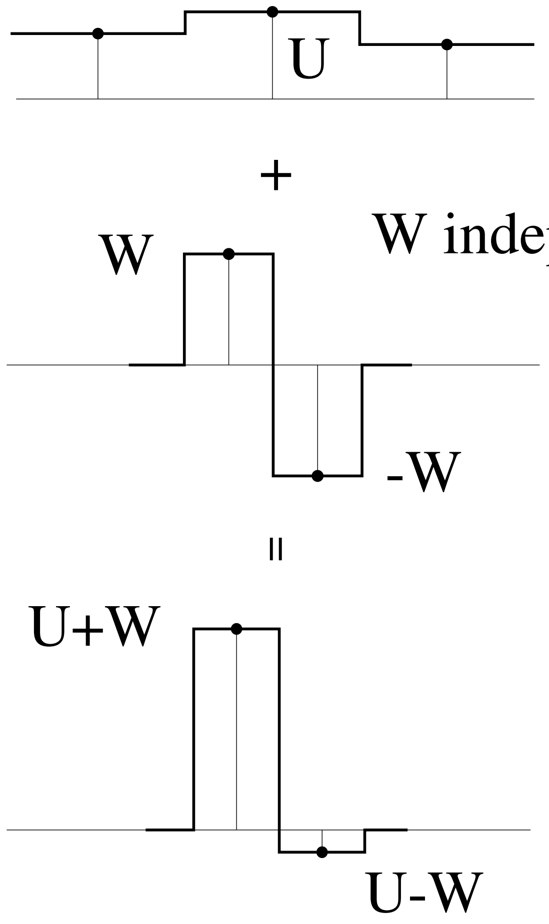
$$\sum_{k=0}^{\infty} r_X(k) = \infty$$

Variance-Time Plot

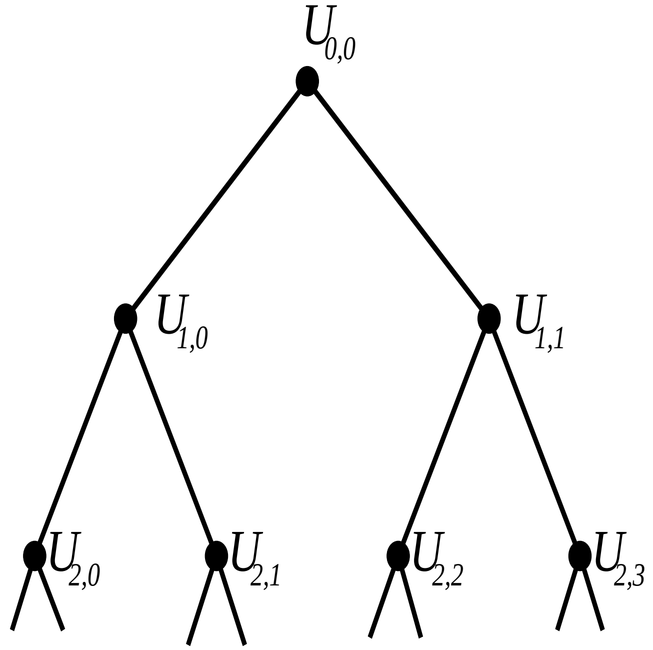



Haar wavelet system

# Additive Multiscale Model



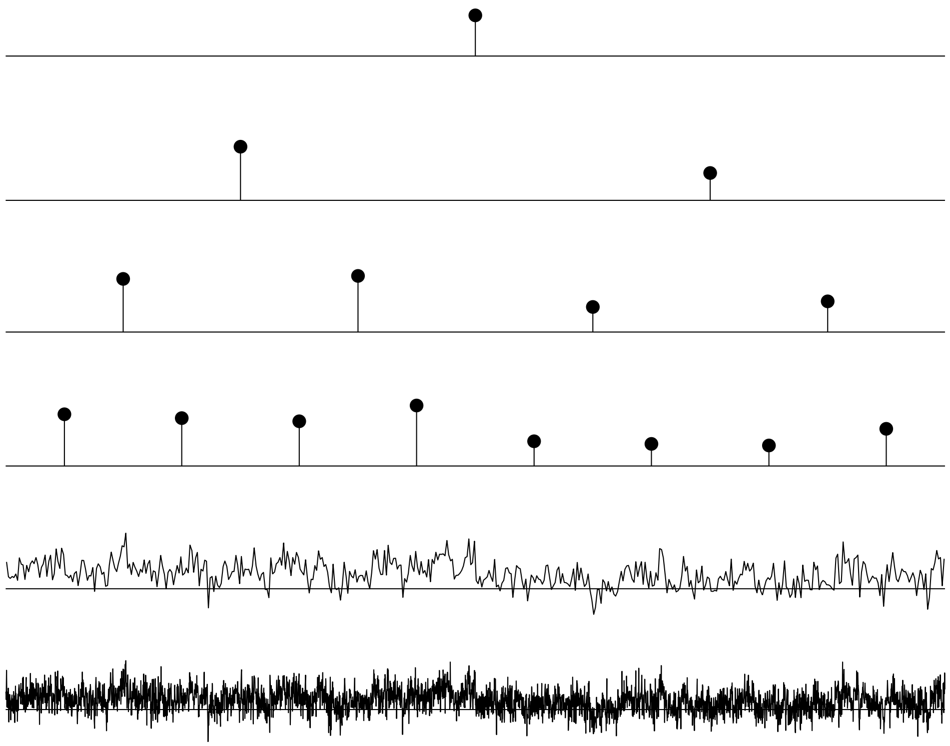

W independent of U




$\mathrm{var}(W_j) \sim 2^{-j(2H-1)} \to$ LRD

# Independent additive Innovations

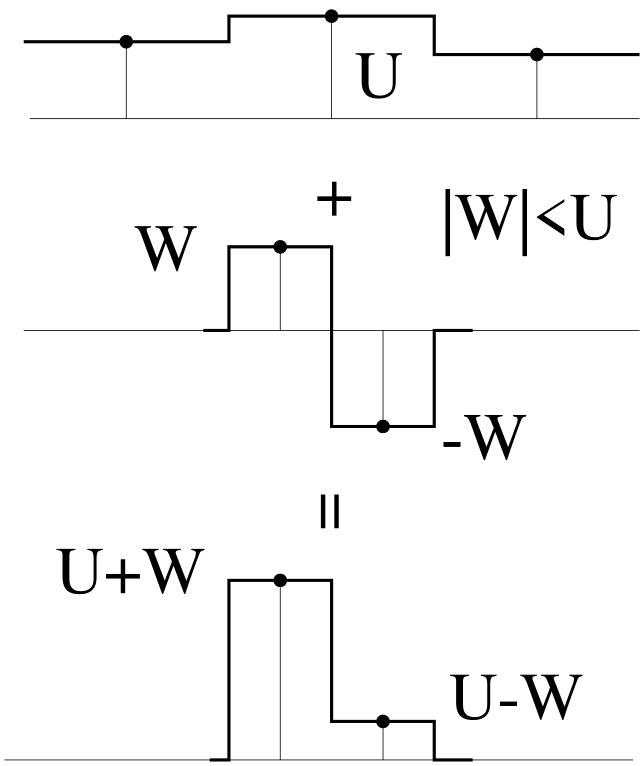
iteration/scale

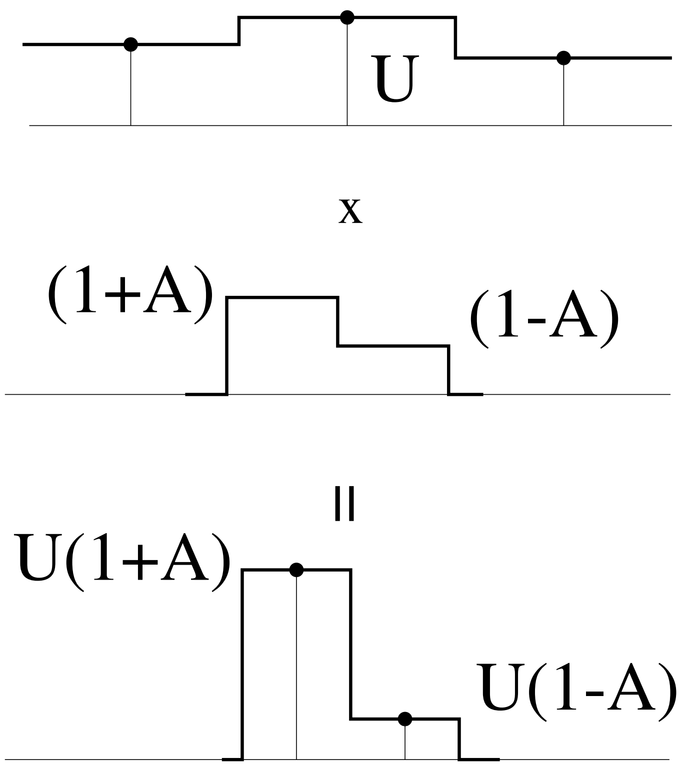



Gaussian LRD noise (close to fGn)

PARSIMONY: $\mathrm{var}(W_j) \sim 2^{-j(2H-1)} \to$ LRD

## Positive Processes through Multiplication

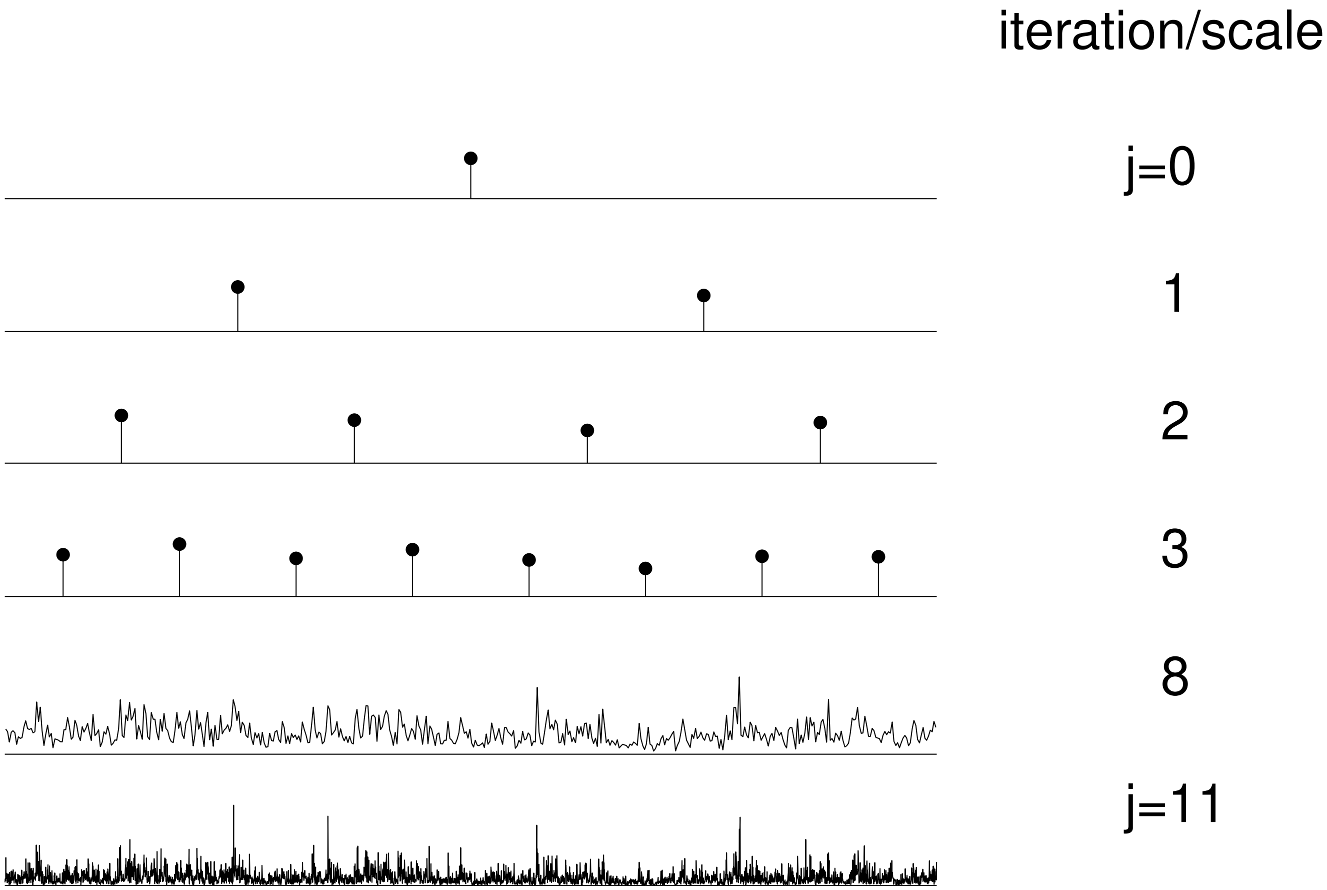



$W_j = A_j \cdot U_j$ with $|A_j| \leq 1$
Dependent additive Innovations

$U_{j+1} = (1 \pm A_j)\, U_j$
Independent multiplicative Innovations

# Independent Multiplicative Innovations

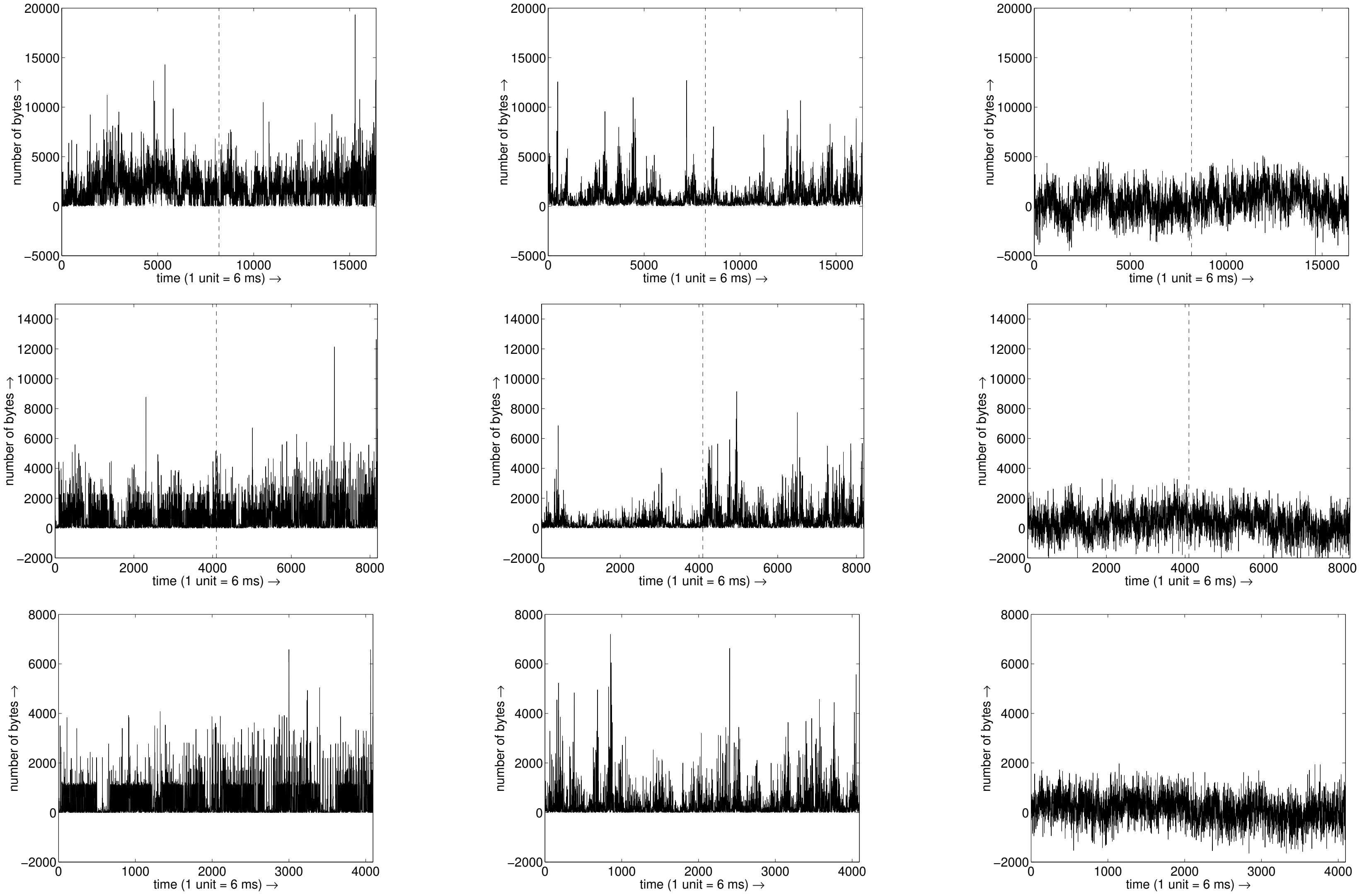



Multiplicative colored Noise, multifractal

PARSIMONY: Variance of multipliers per scale $\rightarrow$ LRD

# SYNTHESIS

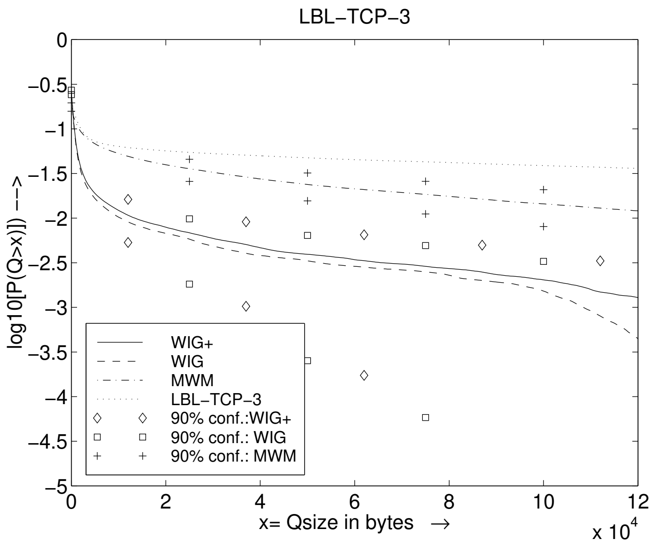
LBL-TCP-3 | Multiplicative | Additive



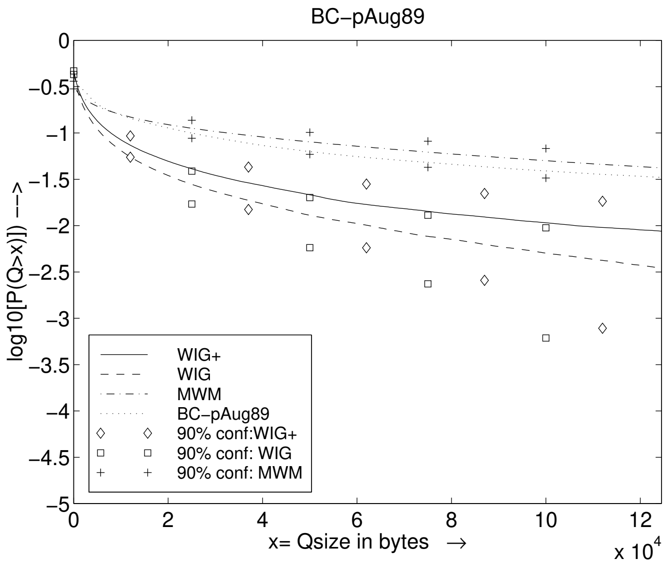
# Single Server Queue: $G$ / $D$ / 1

- Open loop, infinite buffer, $\log P[Q > b]$ vs. $b$

LBL 1994 WAN (TCP)




Bellcore 1989 Ethernet

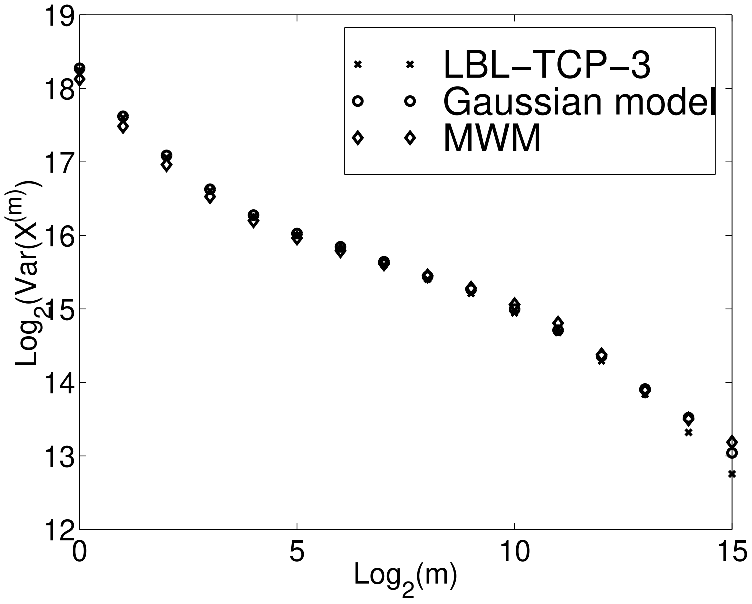



- Weibullian tails (asymptotics) *and* prefactor

- Marginals matter for queueing

# Parsimonious Matching

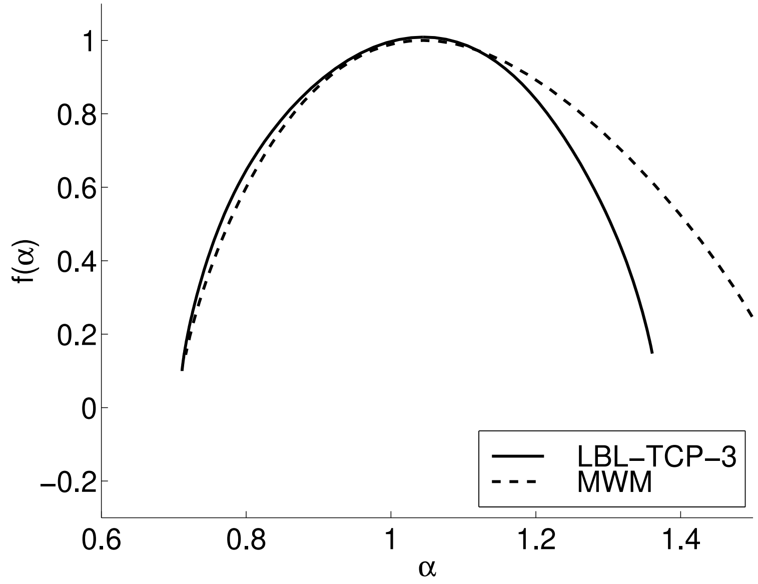
Inserted into model




Variance-time plot

- correlation structure

NOT inserted

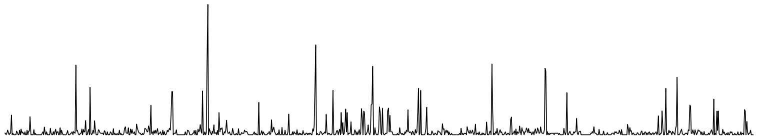



Multifractal spectrum $f(\alpha)$

- higer order dependence
- burstiness
- marginals

# Multifractal Analysis (MFA)

signal

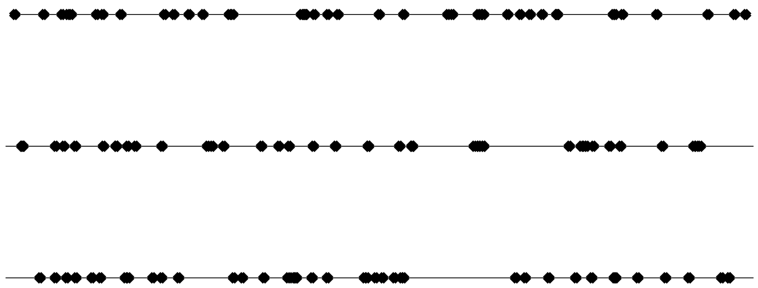


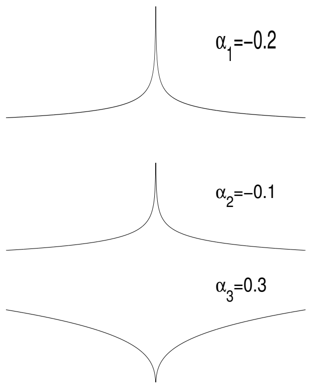
atomic bursts $|t|^{\alpha}$

$\mathcal{E}_{\alpha_1}$

$\mathcal{E}_{\alpha_2}$

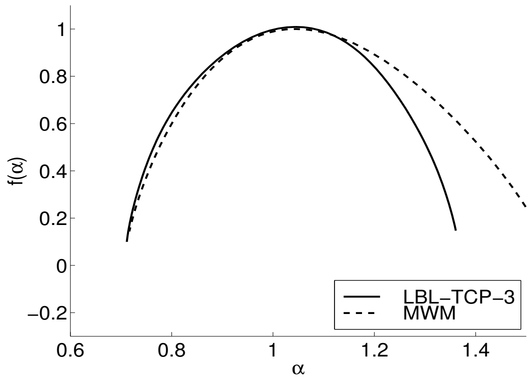
$\mathcal{E}_{\alpha_3}$






MF Spectrum: $f(\alpha) =$ 'size' of $\mathcal{E}_{\alpha}$

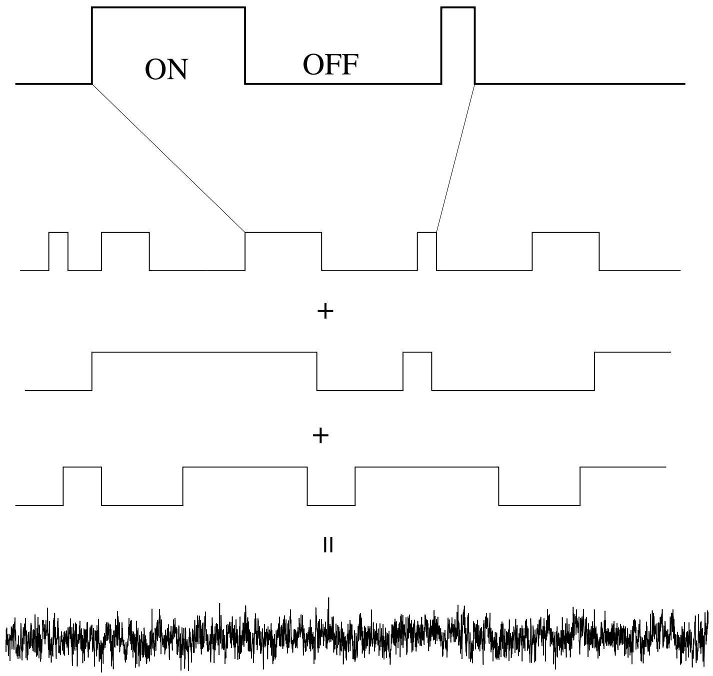
Measure of burstiness



## Why not . . .

| | |
|---|---|
| FARIMA with positive i.i.d. noise | slow, not parismonious<br>little control on marginals |
| Exponentiate colored noise | difficult to pre-warp correlations<br>$\mathbb{E}[e^X] \neq \exp \mathbb{E}[X]$ |
| doubly stochastic Poisson | GOOD IDEA<br>shifts problem to intensity |

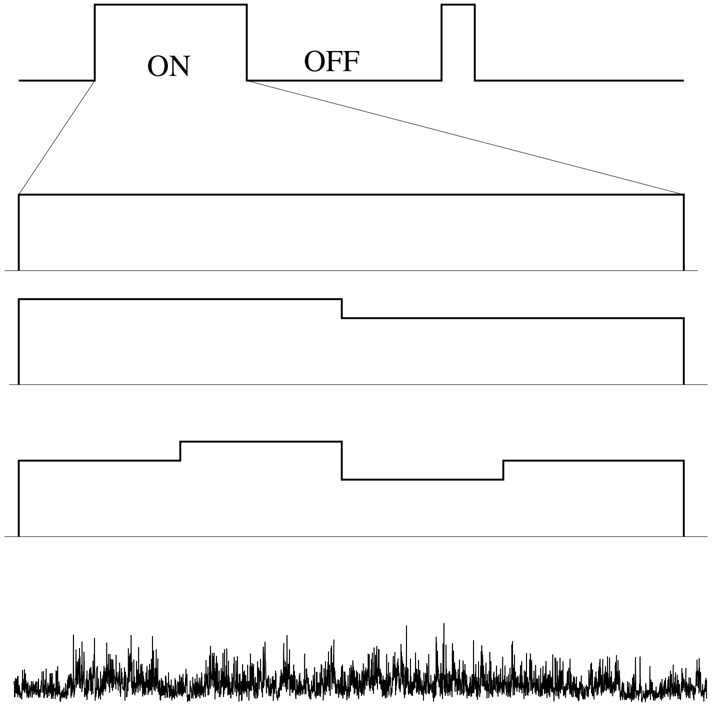
# Understanding the Large Time Scales




Superposition of independent sources

Additive on large time scales : Client behavior

# UNDERSTANDING the Small Time Scales




Partition of traffic load

Multiplicative on small time scales:
TCP connection level
(Feldmann, Gilbert, Willinger)

- Protocol itself (TCP)?
- Multiplexing?
- Topology of internet?

# Multiscale Queuing: Approximations

- $K_r = \sum_{-r+1}^{0} L_i$: aggregate traffic

- Queue was *empty* $\Rightarrow$

$$Q = \sup_{r \in \mathbb{N}} (K_r - rc)$$

- *Dyadic* times scales $\Rightarrow$

$$Q_D = \sup_{m=0,\ldots n} (K_{2^m} - c2^m) \leq Q$$

- *Multiscale* bound: $Pr[Q_D \leq b] =$

$$Pr[K_{2^m} - c2^m \leq b,\, m = 0 \ldots n] \geq \prod_{m=0}^{n} Pr[K_{2^m} \leq b + c2^m]$$

# Multiscale Queuing FORMULA

- $$MSQ(b) := 1 - \prod_{m=0}^{n} Pr[K_{2^m} \le b + c2^m]$$

- Analysis:

$$Pr[Q_D > b] \le MSQ(b)$$

$$Pr[Q_D > b] \le Pr[Q > b]$$

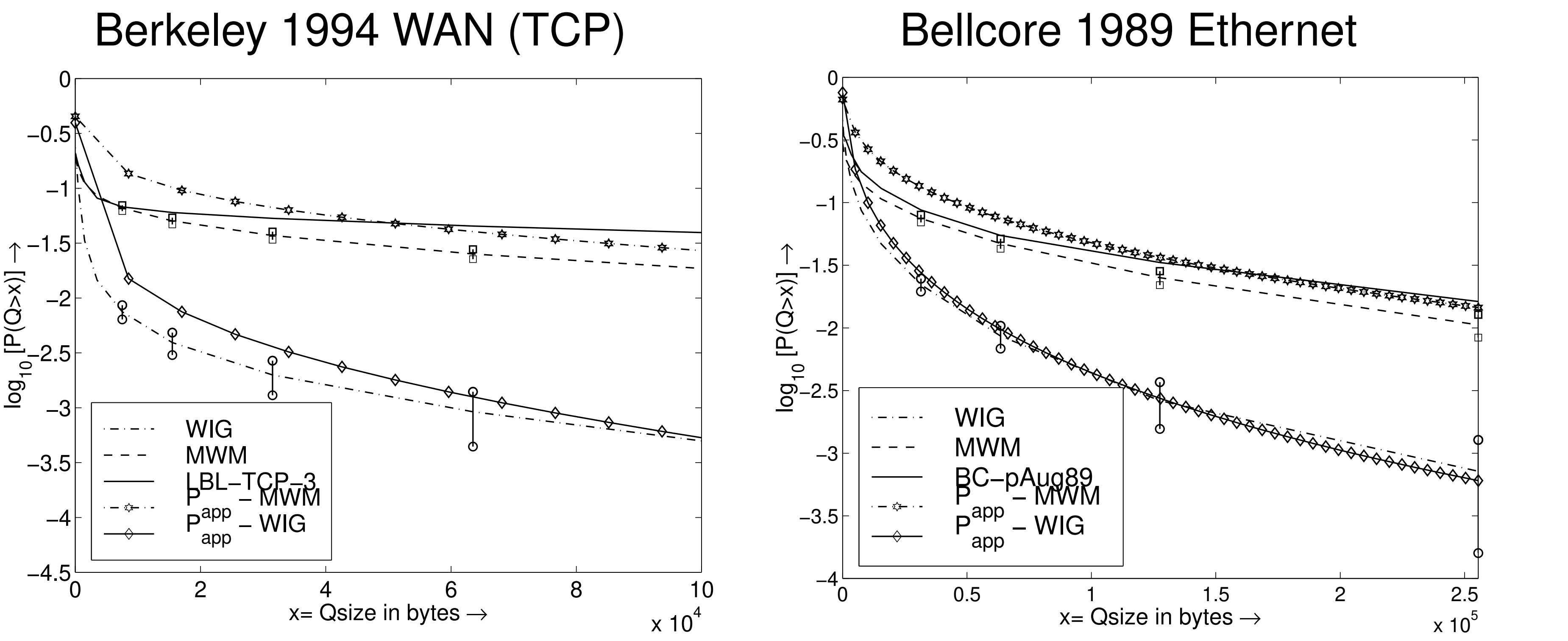
- Simulations for multiscale processes

$$\boxed{Pr[Q > b] \simeq MSQ(b)}$$

- *Dyadic* times scales are “sufficiently independent”
  ... yet representative for all time scales

# Multiscale Queuing of real data traffic

- Multiscale Queuing formula matches real traces (open loop)




- *Marginals* on *all* (dyadic) scales matter

## Lessons and Benefits

- PARSIMONOUS way of modeling bursts
- fast multiplicative SYNTHESIS
- for small time scales THINK ‘partition’, not ‘superposition’
- multiscale QUEUEING FORMULA
- MULTIFRACTAL ANALYSIS: statistical tool to quantize burstiness
- TRIGGERS question:
  what causes multiplicative structure on TCP level

## Where are we headed

- STATIONARY, causal models (Norros, Petteri, VTT)
- Multifractals as INVARIANTS (Knightly, Chengzhi, Rice)
- PREDICTION of bursts (Coates, Ribeiro, Baraniuk, Rice)
- WEB REQUESTS, e-commerce (Almeida, UFMG)

For more details see the publications on our web site www.DSP.RICE.edu, in particular our contributions to SigMetrics 99, Infocomm 2000 and our Technical Report TR 99-08.