Significance in Scale Space

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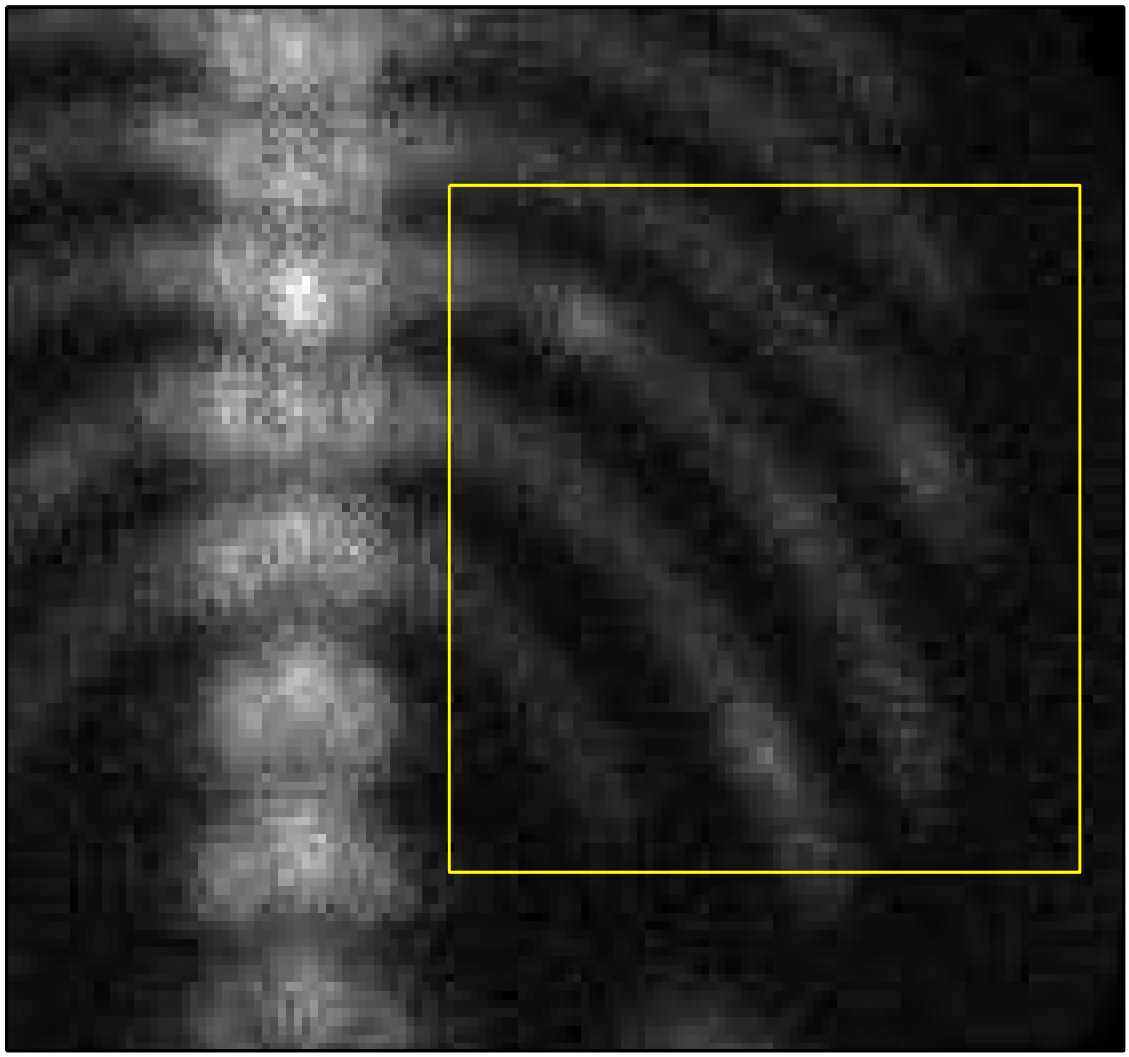
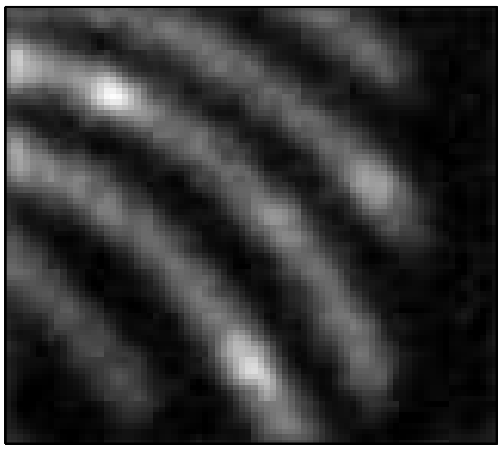
Gamma Camera Phantom Image



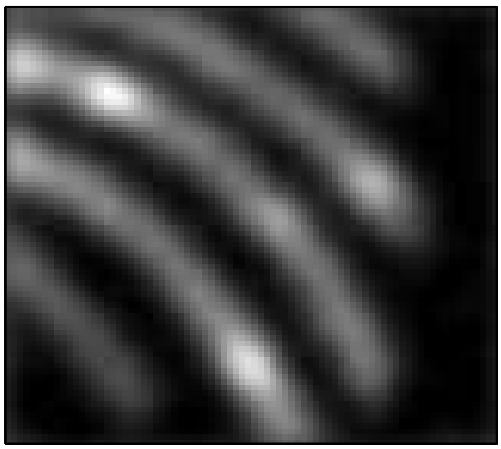
Subimage for later analysis

Gaussian Window Smooths

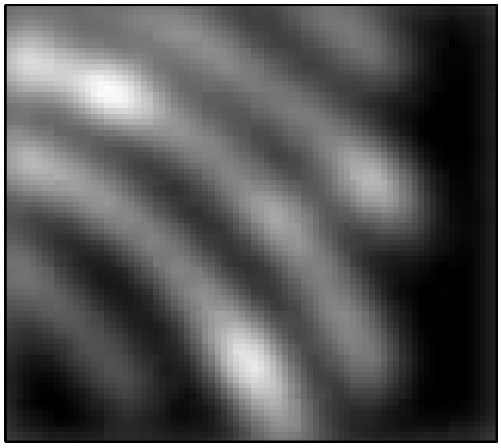
h = 1



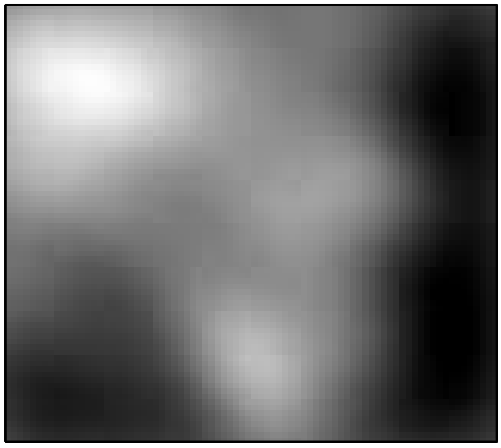
h = 2



h = 4



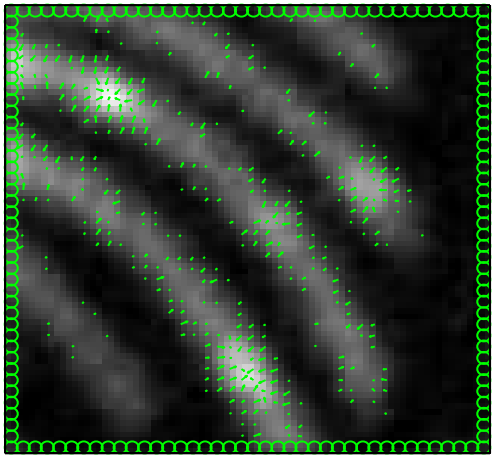
h = 8



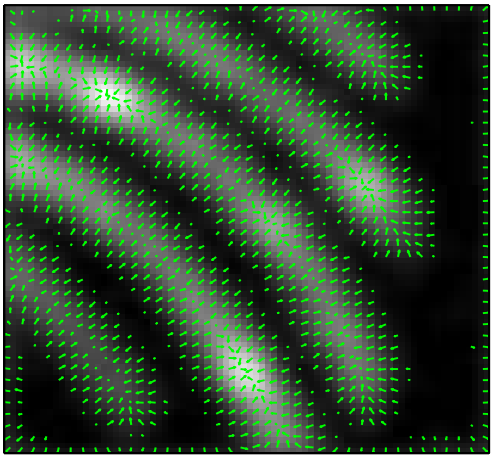
$$\underline{\hat{s}}_h = \underline{K}_h * \underline{Y}$$

Gradient Based S^3

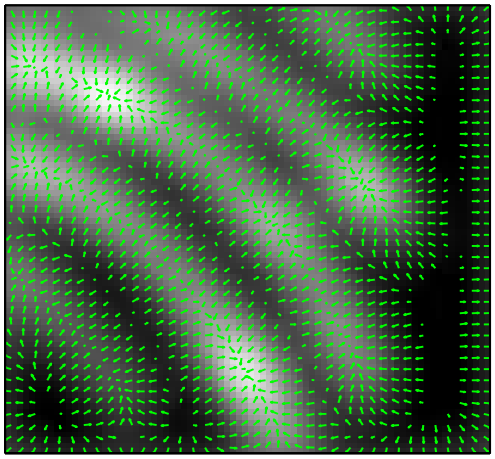
h = 1



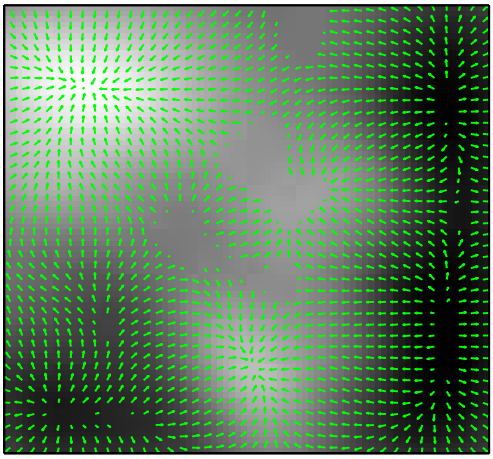
h = 2



h = 4

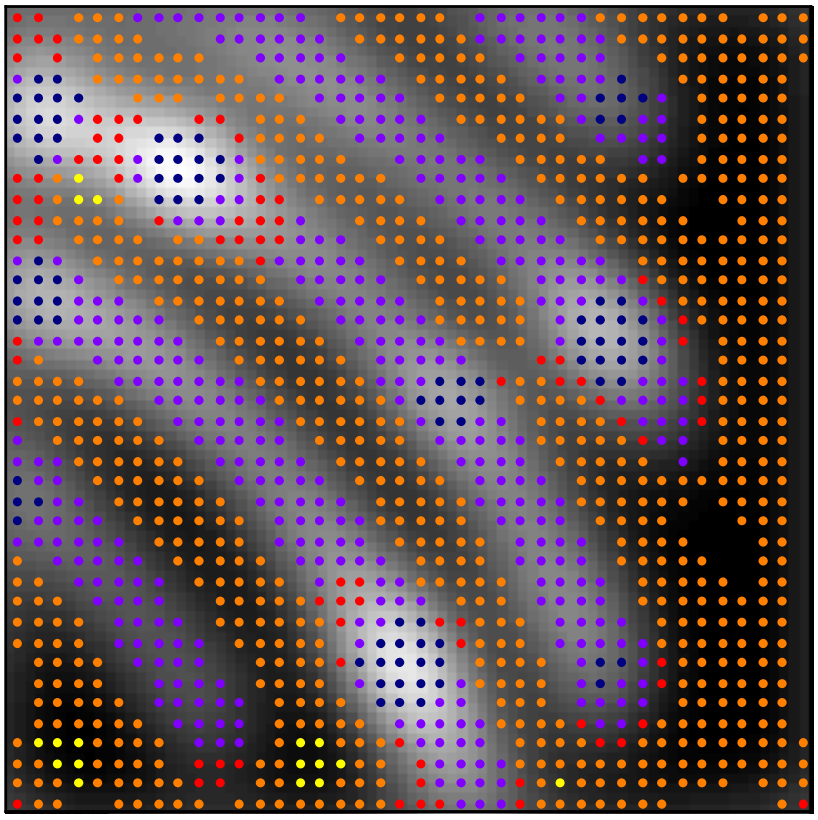


h = 8

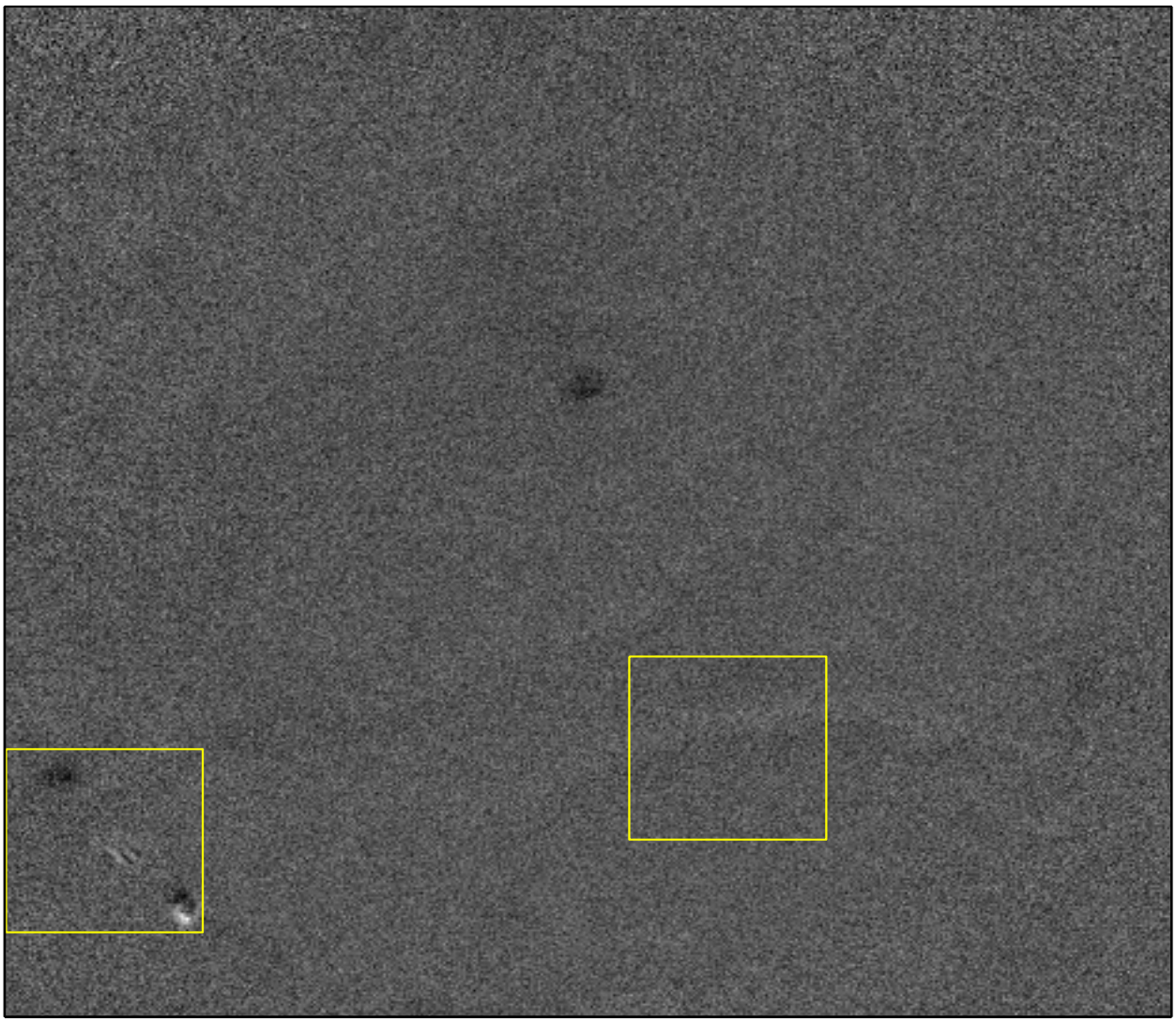


Curvature Based S^3

h = 4



Electrical Activity Image

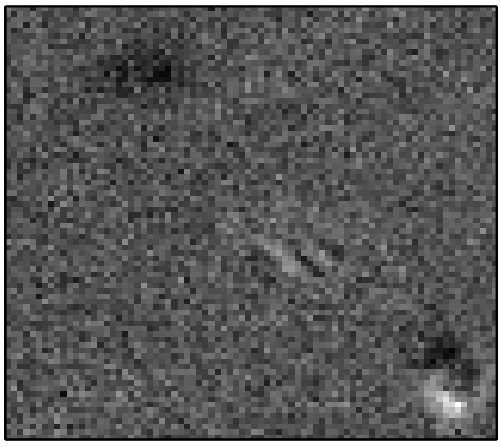


Subimages for later analysis

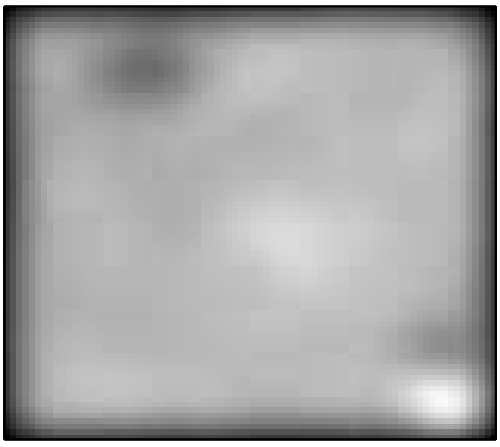
Boundary Effect Handling

Electrical Activity Data

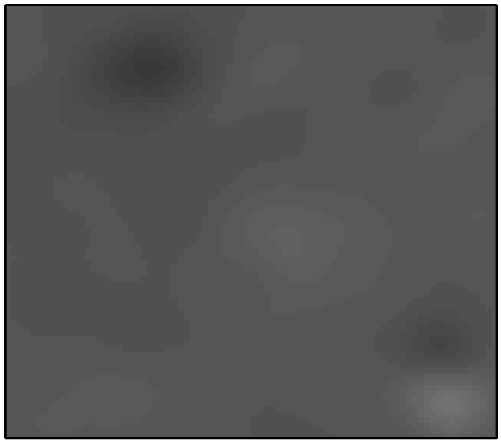
Raw Data



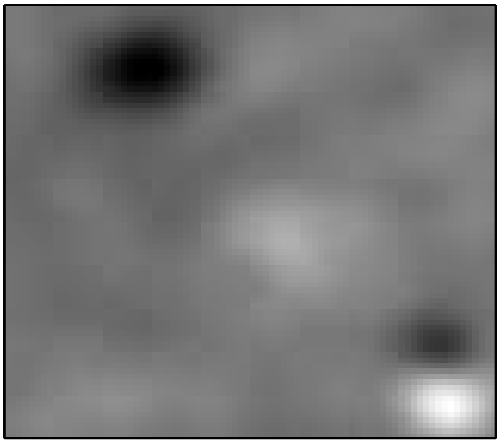
Unadjusted, h = 4



Unscaled, h = 4



Adjusted, h = 4



$$\underline{\hat{s}}_h = A(\underline{Y}) + \underline{K}_h * \left[\underline{Y} - A(\underline{Y})\right]$$

Simultaneous Inference

- Via Effective Sample Size:

$$\underline{ESS} = \left(\underline{K}_h * \underline{1}\right) / K_h(0,0)$$

(number of points in kernel window)

- Number of Independent Averages:

$$l = \frac{n \cdot m}{Avg(\underline{ESS})}$$

- Significance Level Adjustment:

$$\alpha = P\{k - \text{th C.I. not covering}, k = 1,...,l\}$$
$$= 1 - P\{\text{C.I. covers}\}^l = 1 - \left(1 - \alpha'\right)^l$$

Variance Estimation

- Known (e.g. MRI)

- Local Estimate

- Pooled Estimate

Significant Gradient

When gradient “significantly > 0”,

Draw arrow in gradient direction

When arrows too short, use 2 x 2 pixel blocks,

and scale by number significant

Distribution Theory: based on C.L.T. to

Chi-Squared distribution

Significant Curvature

When eigenvalues $\lambda_+ \geq \lambda_-$ of Hessian matrix are "significantly > 0", draw dots as:

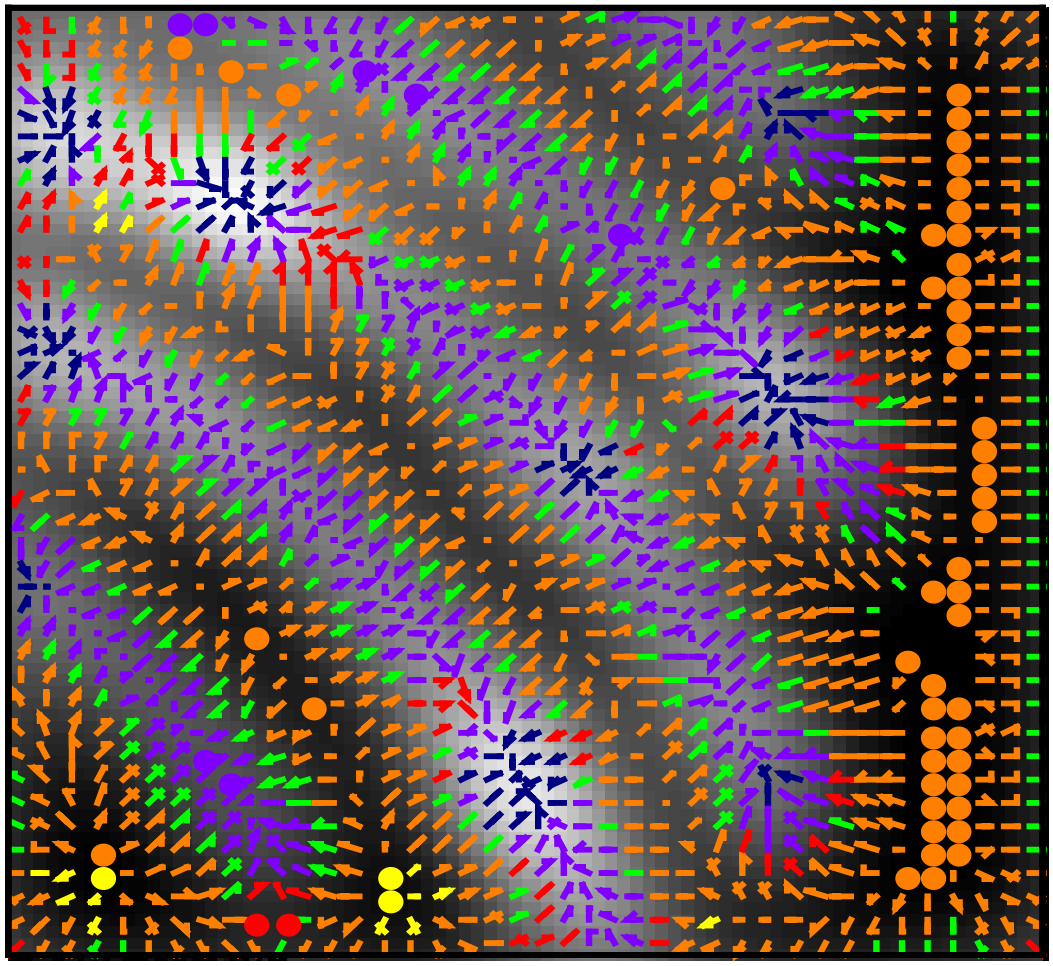
hole	$\lambda_+, \lambda_- > q$
long valley	$\lambda_+ > q, \lvert\lambda_-\rvert < q$
saddle point	$\lambda_+ > q, \lambda_- < -q$
long ridge	$\lvert\lambda_+\rvert < q, \lambda_- < -q$
peak	$\lambda_+, \lambda_- < -q$

(where q is appropriate quantile)

Distribution Theory: based on C.L.T. and simulation

Combining Gradient & Curvature

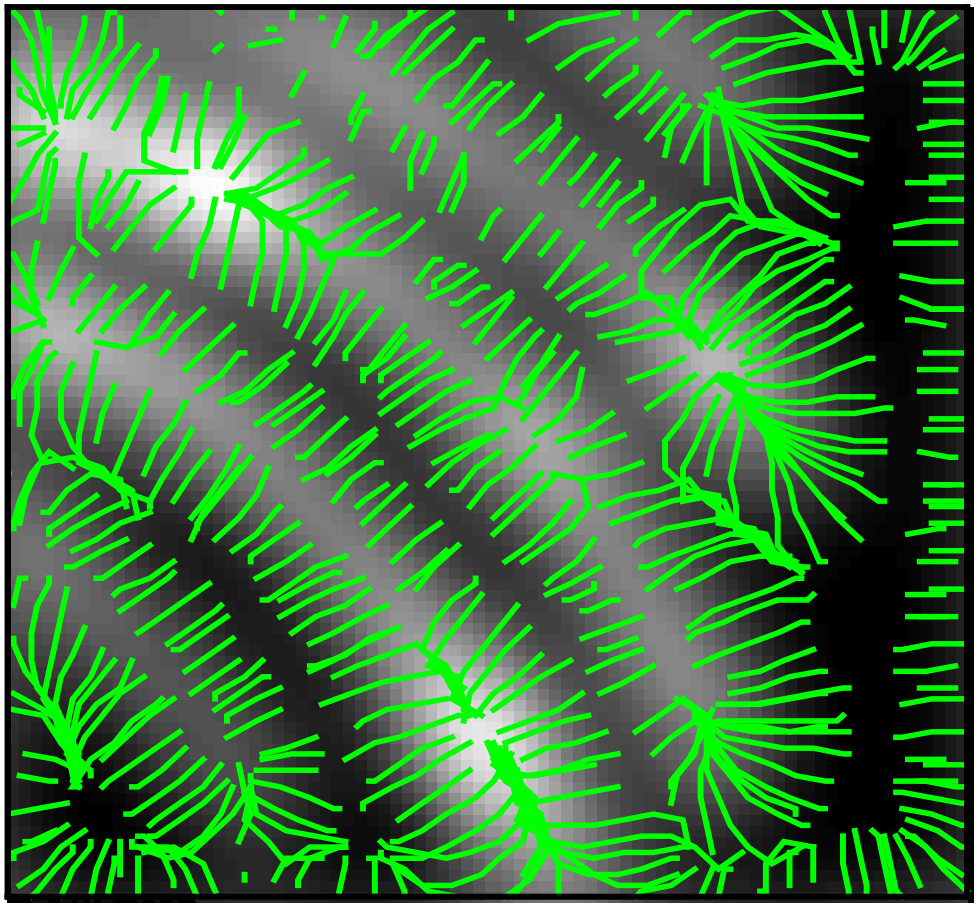
h = 4



Streamlines

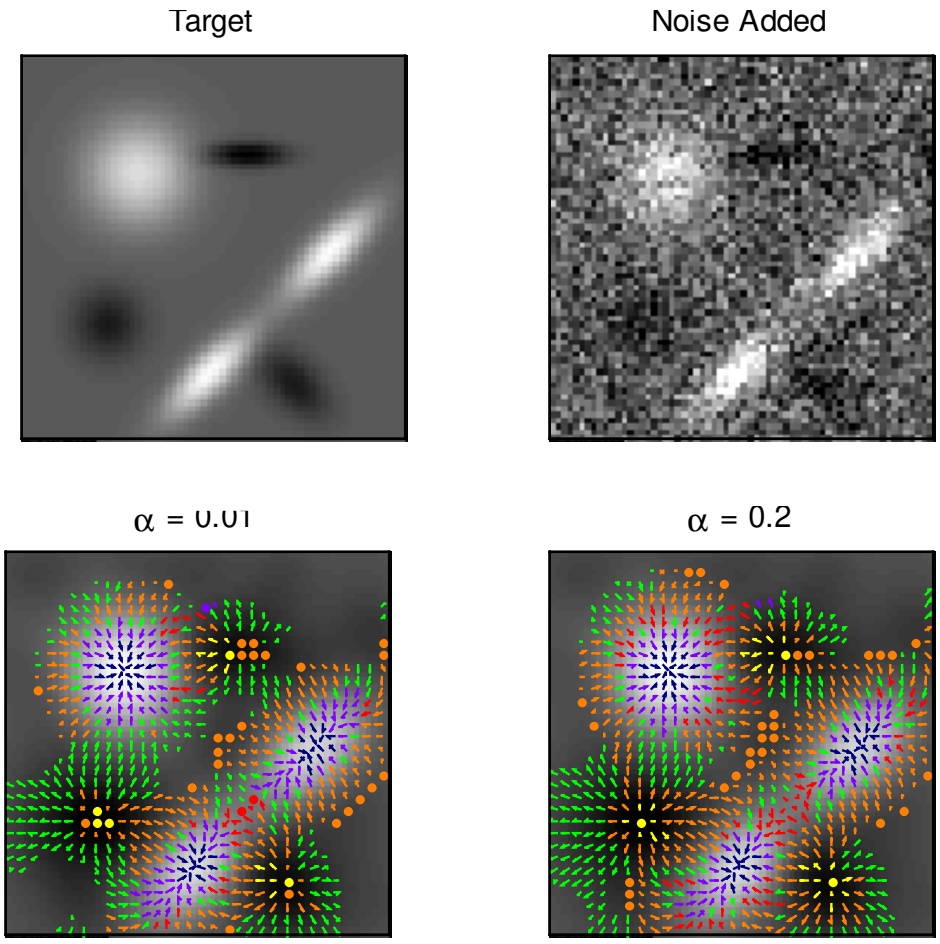
- show gradient directions, where significant

h = 4



Simulations, I

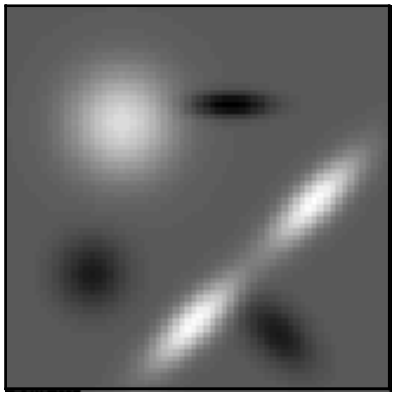
- Effect of significance level α:

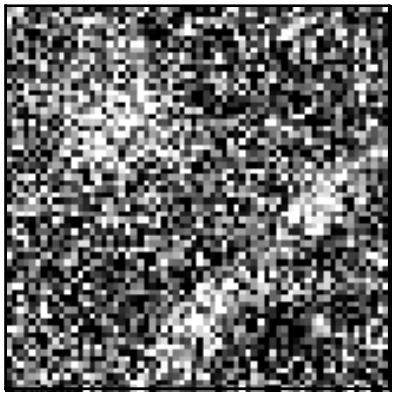


Simulations, II

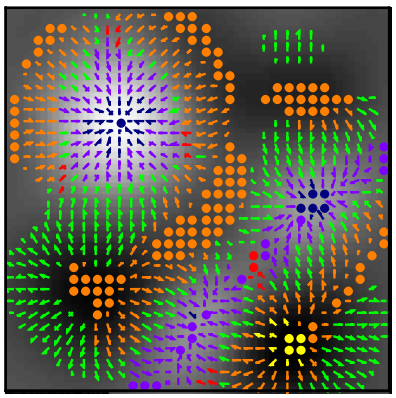
- High Noise Performance:

Target





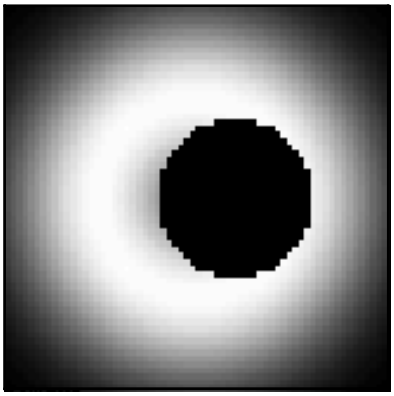
h = 6



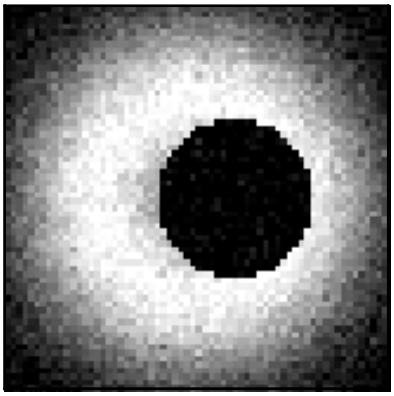
Simulations, III

- Comparison of Methods

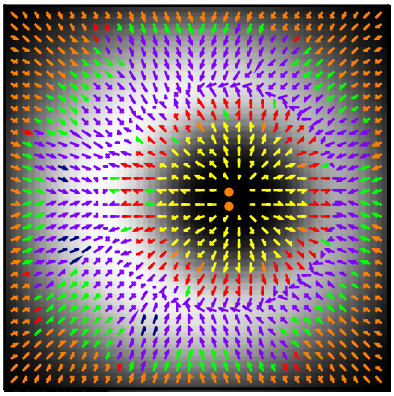
Target



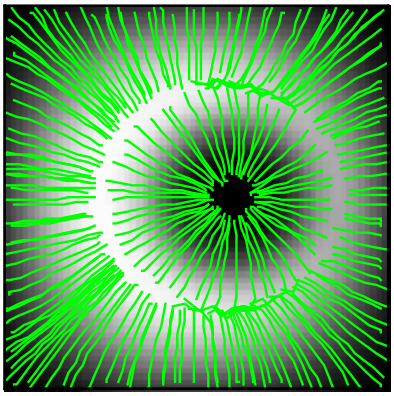
Noise Added



h = 4

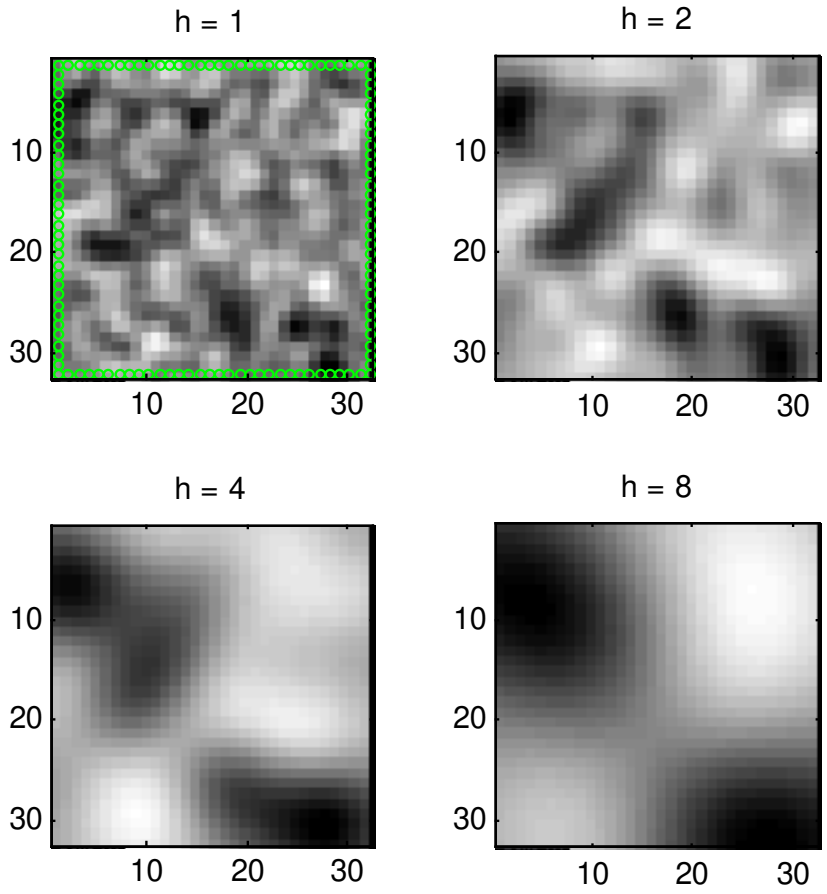


h = 4



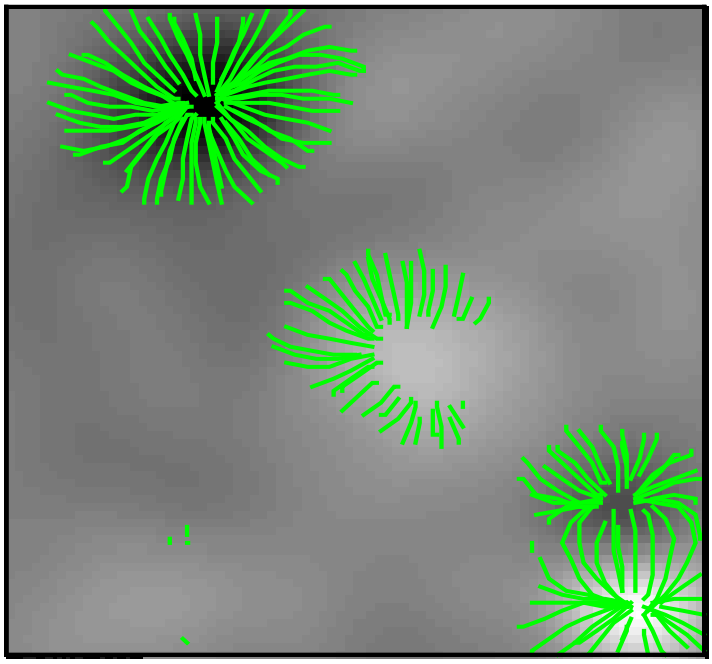
Simulations, IV

- No Signal

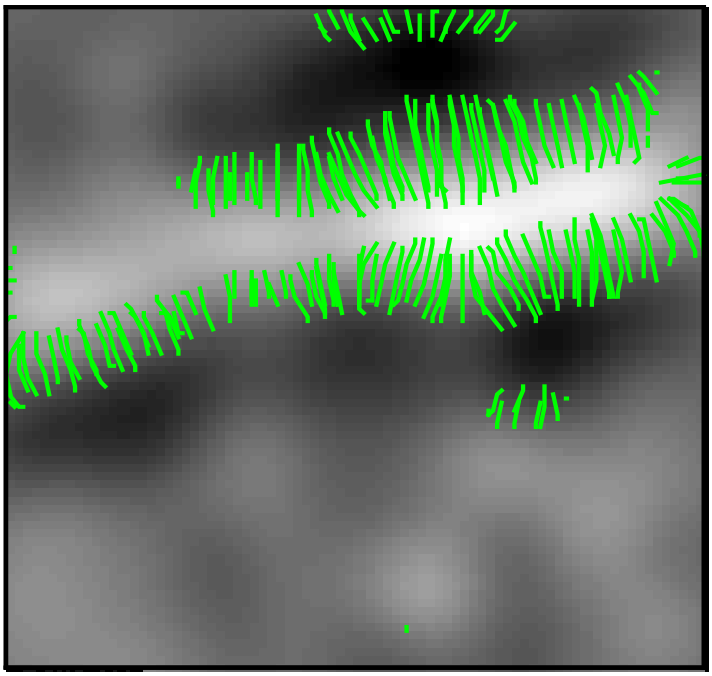


Real Data I, Brain Activity

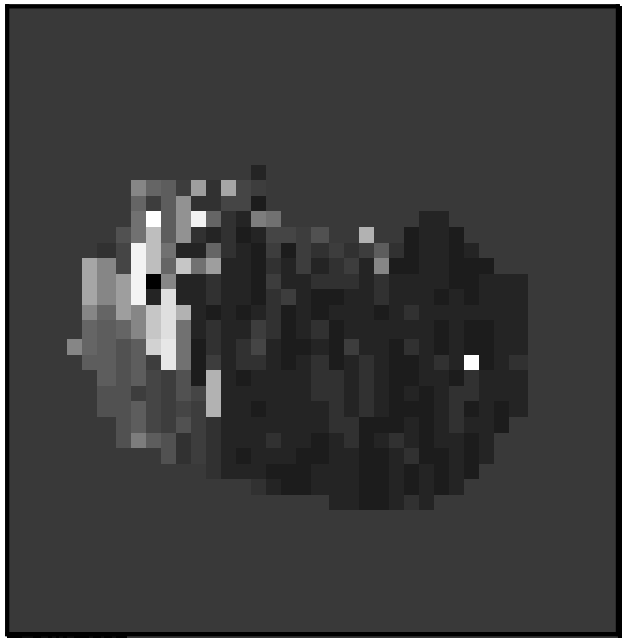
h = 5



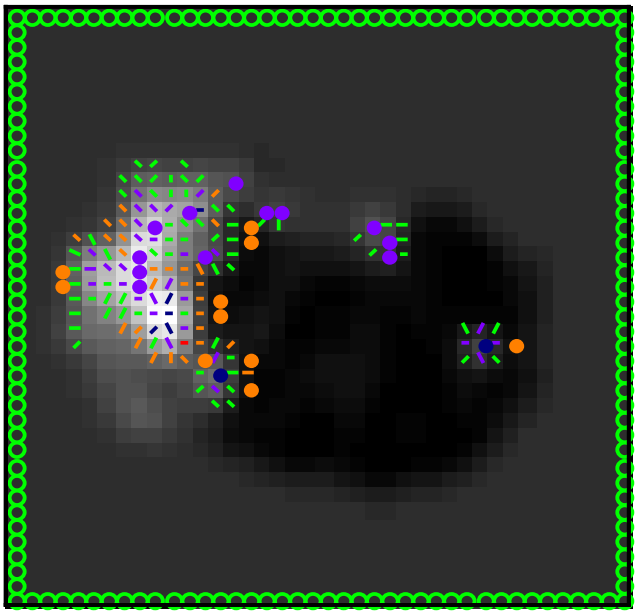
h = 5



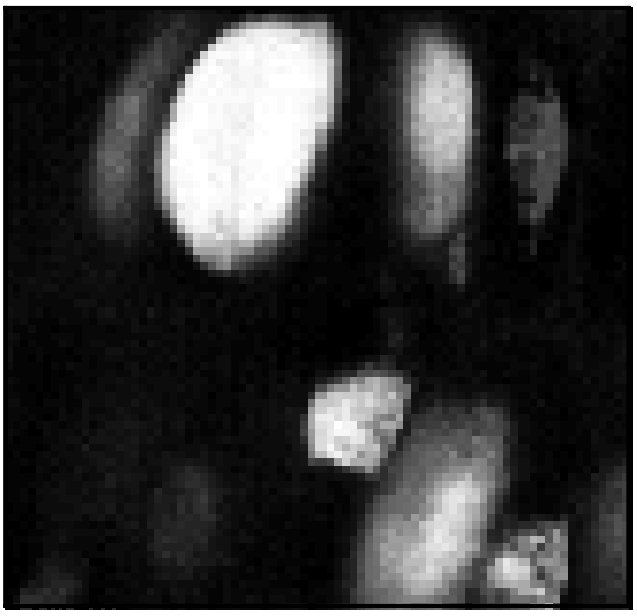
Real Data II, MRI Perfusion



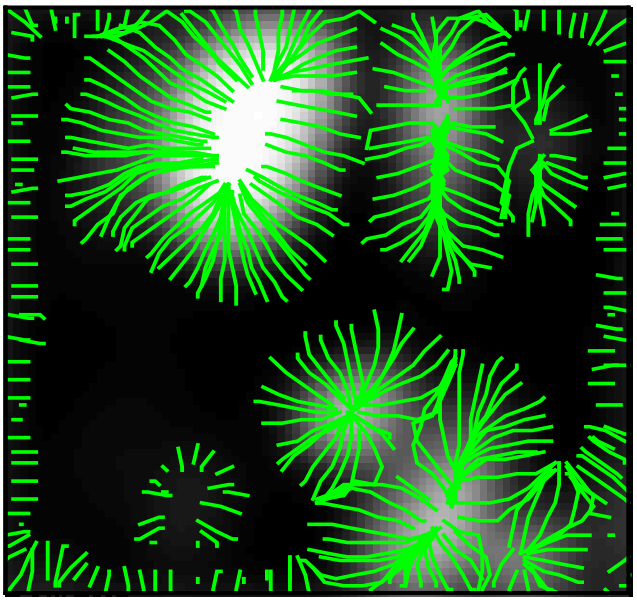
h = 1



Real Data III, Confocal



h = 3



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